Chapter 4

Mathematical Concepts

> If one is master of one thing and understands one thing well, one has at the same time insight into and understanding of many things.
>
> —Van Gogh.

Exhibit 4-1

OBJECTIVES

Upon completion of this chapter the clinician should be able to:

1. Solve basic algebraic equations for *x*, allowing for precision to the number of significant digits
2. Make necessary metric conversions to solve equations and express the answer with the fewest possible non-integer terms

KEY TERMS

accuracy
cubic centimeter
dimension
dimensional unit
precision
quotient
unit of measure
word factor

RATIO EQUATIONS WITH METRICS

Word factors are not easily eliminated from an equation. The equation must be factored to create an expression with the same word factor in the numerator and denominator. This creates an expression divided by itself, which is always a **quotient** of 1, thereby dividing out word factors from the expression.

If the original word factor is in the denominator, factoring will require multiplication. If the word factor is in the numerator, factoring will require division. Remember: multiplication and division must be performed equally on both sides of an equation.

Word factors can only be reduced to 1 when they are divided by an exact duplicate of themselves. Therefore, to manipulate **units of measure** in an equation, word factors should be simplified by conversion to like word factors whenever possible.

DIMENSIONS

A **dimension** is a quality of measure. The metric system has three basic dimensions of concern; length, volume, and mass. Within each dimension there are **dimensional units** of measure. For example, in the dimension of mass, the units of measure include gram, milligram, microgram, and kilogram.

All metric expressions of the same dimension (length, volume, or mass) in an equation should be simplified by conversion to like dimensional units (Exhibit 4-2). The use of like units permits factoring the equation to eliminate word factors.

Mass	kilogram	gram	milligram	microgram	nanogram
Volume	liter	milliliter	deciliter	cubic centimeter	
Length	kilometer	meter	centimeter	millimeter	micrometer

Exhibit 4-2 Like Measurements

For example, factors ending in the dimension unit "gram"—such as milligram, microgram, kilogram, or gram—are all units of mass. These should, for medicinal administration, all be converted to the same dimensional unit, such as milligram.

This dimensional conversion rule applies to all metric dimensions used in equations. In the metric dimension of volume, the dimensional units are liter and milliliter. A **cubic centimeter** (cc) is a measurement of area equal to a milliliter. Milliliters and cubic centimeters are frequently used interchangeably. However, when discussing volume (rather than cubic area), milliliter is the most correct expression.

Conversions may be made between units *within* a dimension, such as mass (grams) to mass (milligrams). Conversions cannot be directly made *between* dimensions, such as from mass (grams) to volume (milliliters).

> All metric units of the same dimension should be converted to like units.
>
> Conversions cannot be made directly between different dimensions, such as mass (grams) to volume (milliliters).

Exhibit 4-3 Dimensional Conversion Rule

TARGET SELECTION

Once the clinician has decided to simplify an equation with conversions between dimensional units, the target, or unit to which the conversion is to be made, must be selected. For example, does one convert from milligrams to grams or from grams to milligrams? Selecting a target is not critical to the operation. Any dimensional conversion that is mathematically sound will permit solution of the equation. Expressing measurements in a unit that allows the smallest whole number or mixed number (a whole number and a fraction) is helpful when deciding whether to convert to larger or smaller units of measure. Smaller whole numbers are easier to multiply or divide than large numbers. For example, *7.2 kilograms* is easier to work with than 7,200 mg. or 7,200,000 mcg. Also as a contrast, 7.2 mg is easier to work with than 0.0072 kg. There are two guiding considerations in selecting the simplest target dimensional unit.

> Any dimensional conversion that is mathematically sound will permit solution of an equation.
>
> However, units that allow the clinician to discuss the calculation in integers are preferred.

Exhibit 4-4 Evaluation of the Most Correct Metric Terms

First, in what unit is the medication measured? If possible, all conversions should be made to the unit in which the medication is measured. If a medication is measured in milligrams, the logical target selection is milligrams, and all dimensional units of mass will be converted to milligrams.

Second, which dimensional units will permit the most simplified mathematical operations? If conversion to grams will require arithmetic to the third decimal place, one may wish to work in milligrams to simplify operations.

Exhibit 4-5 is an equation that requires dimensional analysis and conversion of metric word factors. The clinician must convert kilograms and milligrams to a common dimensional unit. In Exhibit 4-5, converting kilograms to milligrams results in a number in the millions.

In the following equation, a conversion must be made to simplify the use of dimensional units.

$\frac{x}{5\text{ kg}} = \frac{100\text{ ml}}{5{,}000\text{ mg}}$	Kilograms and milligrams are both units of mass and must be converted to a common dimensional unit to form a common denominator.
Convert kilograms to milligrams.	
$5\text{ kg} = ?\text{ mg}$	Conversion from kilograms to milligrams is desired.
$5\text{ kg} = \left(\frac{1{,}000{,}000\text{ mg}}{1\text{ kg}}\right) = y\text{ mg}$	The unit conversion factor substitution method is used.
$5\text{ kg} = 5{,}000{,}000\text{ mg}$	The answer is 5,000,000 mg.
Now the original equation may be expressed:	
$\frac{x}{5{,}000{,}000\text{ mg}} = \frac{100\text{ ml}}{5{,}000\text{ mg}}$	Substitute 5,000,000 mg for 5 kg in the equation. Now the equation may be solved for x because it has a common denominator.
$x = 100{,}000\text{ ml or } 100\text{ L}$	

Exhibit 4-5 Example Unit Conversion

Conversions that do not simplify the mathematical operation as much as possible still yield correct results. Larger numbers are more difficult to use mathematically. The additional difficulty increases the possibility of error.

A more effective dimensional conversion may be to convert both the kilograms and the milligrams to grams, as in Exhibit 4-6.

Given the following problem, solve for *x*.

$$\frac{x}{5\text{ kg}} = \frac{100\text{ ml}}{5{,}000\text{ mg}}$$

Kilograms and milligrams are both units of mass and must be converted to a common dimensional unit and a common denominator.

Convert kilograms and milligrams to grams.

Problem 1: $5\text{ kg} = y\text{ g}$

Problem 2: $5{,}000\text{ mg} = z\text{ g}$

Convert from kilograms to grams. Convert from milligrams to grams.

$$5\text{ kg}\left(\frac{1{,}000\text{ g}}{1\text{ kg}}\right) = y\text{ g}$$

The unit conversion factor substitution method is used.

$$5\text{ kg} = 5{,}000\text{ g}$$

The answer is 5,000 g.

A similar operation is used to convert the 5,000 mg to 5 g. Now the original equation may be solved with a common denominator.

$$\frac{x}{5{,}000\text{ g}} = \frac{100\text{ ml}}{5\text{ g}}$$

Now the equation may be solved for *x*.

Kilograms and milligrams are both units of mass and must be converted to a common dimensional unit and a common denominator.

$$\frac{x}{5\text{ kg}} = \frac{100\text{ ml}}{5{,}000\text{ mg}}$$

A similar operation is used to convert the 5,000 mg to 5 g. Now the original equation has units expressed in grams and may be solved using a common denominator.

$$\frac{x}{5{,}000\text{ g}} = \frac{100\text{ ml}}{5\text{ g}}$$

Now the equation may be solved for *x*.

$$\cancel{5{,}000\text{ g}}\left(\frac{x}{\cancel{5{,}000\text{ g}}}\right) = \overset{1{,}000}{\cancel{5{,}000\text{ g}}}\left(\frac{100\text{ ml}}{\cancel{5\text{ g}}}\right)$$

$$x = 100{,}000\text{ ml}$$

$$x = 100\text{ L}$$

To reduce a large metric number to a more workable number, we convert milliliters to liters using a unit conversion factor again.

Exhibit 4-6 Example Metric Conversions

Conversions must be made in Exhibit 4-5, Exhibit 4-6, and Exhibit 4-7 to set up a common denominator so the problem can be solved. This will simplify the use of dimensional units.

$$\frac{x}{5\text{ g}} = \frac{1\text{ ml}}{250\text{ mg}}$$

To solve, units in the denominators must be the same denomination. So we can either convert from milligrams to grams or from grams to milligrams. Converting to milligrams will give us whole numbers to work with, while converting to grams will have us working with a decimal number. Either conversion will work, but the author prefers to work with whole numbers, so we will convert to milligrams to give us a common denominator. (The unit conversion operation is not shown in this example.)

$$\frac{x}{5{,}000\text{ mg}} = \frac{1\text{ ml}}{250\text{ mg}}$$

$$\overset{1}{\cancel{5{,}000\text{ mg}}}\left(\frac{x}{\underset{1}{\cancel{5{,}000\text{ mg}}}}\right) = \left(\frac{1\text{ ml}}{\underset{1}{\cancel{250\text{ mg}}}}\right)\overset{20}{\cancel{5{,}000\text{ mg}}}$$

$x = 20$ ml

Exhibit 4-7 Example Metric Conversion with Unit Conversion Factor Substitution

60 min/hr

100¢/4 quarters

100¢/$1

52 weeks/year

4 quarters/$1

2.2 lb/kg

12/1 dozen

1,000 ml/L

16 oz/1 lb

Exhibit 4-8 Example Unit Conversion Factors

ACCURACY

Scientists, including health care professionals, make a distinction between **accuracy** and precision. Accuracy is to be free from error, but not necessarily exact. It refers to how closely a measured or estimated value agrees with the correct value. An approximation may be accurate. Patients rarely appreciate having their medication doses approximated.

PRECISION

Precision is accurate, correct, and as exact as possible. The amount of exactness is determined by the exactness of the other numbers used in the mathematical operation. For example, if an order for medication reads "give 1 mg per kilogram," the closest precision is a whole kilogram of patient weight, or in the case of a neonate or a premature infant, the order may even be given per unit of body weight in grams. Doses for neonates must be very precise. It is not possible to be more precise than the numbers in the initial problem. A tenth of a kilogram or one hundredth of a kilogram in this problem is not significant.

SIGNIFICANT DIGITS

Some digits are exact (or precise) numbers that are known to be absolutely accurate. These include scientific constants, such as the speed of light. Other numbers are accurate and only precise to the number of significant digits. The measurements on medication labels are exact.

Significant digits are numbers obtained by measurement and believed to be correct by the clinician making the measurement. The patient's weight is a number limited by the precision of the instrument used to weigh the patient. For example, a scale that measures in whole kilograms cannot be used to determine weight more precisely than a whole kilogram. It cannot precisely determine the weight to hundredths of a kilogram.

Any mathematical operation can only be as precise as the least precise factor in it.

Exhibit 4-9 Rule of Significant Digits

Rules of significant digits:

1. Leading or trailing zeros that hold decimal places are not significant digits. For example, 0.025 has only two significant digits. Also 0.205 has 3 significant digits because the middle zero is neither a leading or trailing zero.
2. In multiplication and division, a precise answer can have significant digits no more precise than the factor with least number of significant digits used in the operation.
3. In addition and subtraction, the last digit retained in the answer is determined by the least precise digit.

For an example equation using significant numbers, see Exhibit 4-10.

Consider the equation 1.2 ∗ 3.111113. It will give an answer significant only to the one decimal point because 1.2 is the limiting number of significant digits, and it has two significant digits. The other number, 3.111113, has seven significant figures, but the answer can only be as significant as the least precise number used in deriving it.

$1.2 * 3.111113 = 3.7333356$

The answer will, for scientific and medical use, be rounded off to 3.7. Only one decimal point is significant because the least precise number in the problem was 1.2 and it has one place after the decimal point.

Exhibit 4-10 Significant Digits in Calculations

Rounding Off

The number of significant digits determines the number of decimal places to which the problem can be precisely calculated. A problem is calculated to one more decimal place than the number of significant digits and then rounded off.

Rounding off is performed by evaluating the last decimal place. If it is five or greater, the preceding decimal place is rounded up to the next digit. If it is less than five, the preceding decimal place is left as it is. The last decimal place is then dropped from the number, as shown in Exhibit 4-11.

3.246	is rounded to	3.25
3.244	is rounded to	3.24
4.26	is rounded to	4.3
4.24	is rounded to	4.2
3.54	is rounded to	3.5

Exhibit 4-11 Rounding Off

Most Appropriate Expression

Metric expressions may be interchanged without loss of precision. For example: 1,500 mg and 1.5 g are the same quantity. (Note: 1,505 mg is more precise than 1,500 mg.) These two expressions are equally correct and precise. However, one is more appropriate than the other. The most appropriate expression is 1.5 g.

Two considerations guide the clinician in determining which expression is the most appropriate:

1. The expression with the lowest whole numbers is appropriate. For example, 5 g is more appropriate than 5,000 mg or 5,000,000 mcg.
2. The solution to an expression cannot be more precise than the initial number of significant digits.

Review Problems

Solve the following equations for x. Make any necessary dimensional conversions.

1. $\frac{4x}{2\text{ L}} = \frac{4\text{ mg}}{2\text{ ml}}$

2. $\frac{4x}{2\text{ mg}} = \frac{4\text{ ml}}{2\text{ g}}$

3. $\frac{x}{1\text{ L}} = \frac{1\text{ mg}}{1\text{ ml}}$

4. $\frac{x}{1\text{ ml}} = \frac{1\text{ g}}{2\text{ L}}$

5. $\frac{x}{110\text{ lb}} = \frac{1\text{ mg}}{1\text{ kg}}$

6. $\frac{x}{1\text{ kg}} = \frac{10\text{ mg}}{1\text{ lb}}$

7. $\frac{x}{50\text{ mg}} = \frac{110\text{ lb}}{1\text{ kg}}$

8. $\frac{x}{10\text{ mg}} = \frac{1\text{ kg}}{1\text{ lb}}$

9. $\frac{x}{1\text{ mg}} = \frac{1\text{ ml}}{1\text{ L}}$

10. $\frac{x}{1\text{ g}} = \frac{1\text{ ml}}{2\text{ L}}$

11. $\frac{x}{50\text{ mg}} = \frac{5\text{ ml}}{1\text{ g}}$

12. $\frac{x}{50\text{ mg}} = \frac{1\text{ ml}}{50\text{ mcg}}$

13. $\frac{x}{5\text{ ml}} = \frac{50\text{ mg}}{1\text{ g}}$

14. $\frac{x}{1\text{ ml}} = \frac{50\text{ mg}}{50\text{ mcg}}$

15. $\frac{x}{5\text{ g}} = \frac{5\text{ ml}}{250\text{ g}}$

16. $\frac{x}{1\text{ g}} = \frac{1\text{ tablet}}{250\text{ mg}}$

17. $\frac{x}{5\text{ ml}} = \frac{5\text{ g}}{250\text{ mg}}$

18. $\frac{x}{1\text{ tablet}} = \frac{1\text{ g}}{250\text{ mg}}$

19. $\frac{x}{0.5\text{ mg}} = \frac{1\text{ ml}}{1\text{ mg}}$

20. $\frac{x}{500\text{ mg}} = \frac{10\text{ ml}}{1\text{ g}}$

21. $\frac{x}{1\text{ ml}} = \frac{0.5\text{ mg}}{1\text{ mg}}$

22. $\frac{x}{10\text{ ml}} = \frac{500\text{ mg}}{1\text{ g}}$

23. $\dfrac{x}{30 \text{ ml}} = \dfrac{60 \text{ gtt}}{1 \text{ ml}}$

24. $\dfrac{x}{1 \text{ min}} = \dfrac{120 \text{ ml}}{1 \text{ hr}}$

25. $\dfrac{x}{60 \text{ gtt}} = \dfrac{30 \text{ ml}}{1 \text{ ml}}$

26. $\dfrac{x}{120 \text{ ml}} = \dfrac{1 \text{ min}}{1 \text{ hr}}$

27. $\dfrac{x}{90\%} = \dfrac{10 \text{ ml}}{100\%}$

28. $\dfrac{x}{33.3\%} = \dfrac{333 \text{ mg}}{100\%}$

29. $\dfrac{x}{10 \text{ ml}} = \dfrac{90\%}{100\%}$

30. $\dfrac{x}{333 \text{ mg}} = \dfrac{33.3\%}{100\%}$

31. $\dfrac{x}{15 \text{ mg}} = \dfrac{1 \text{ ml}}{1 \text{ mg}}$

32. $\dfrac{x}{180 \text{ ml}} = \dfrac{10 \text{ gtt}}{1 \text{ ml}}$

33. $\dfrac{x}{1 \text{ ml}} = \dfrac{15 \text{ mg}}{1 \text{ mg}}$

34. $\dfrac{x}{110 \text{ lb}} = \dfrac{1 \text{ ml}}{8 \text{ oz}}$

35. $\dfrac{x}{1 \text{ min}} = \dfrac{60 \text{ ml}}{1 \text{ hr}}$

36. $\dfrac{x}{1 \text{ min}} = \dfrac{1{,}440 \text{ ml}}{1 \text{ day}}$

37. $\dfrac{x}{5 \text{ ml}} = \dfrac{40 \text{ mg}}{1 \text{ L}}$

38. $\dfrac{x}{5 \text{ min}} = \dfrac{4 \text{ mg}}{1 \text{ hr } 40 \text{ min}}$

39. $\dfrac{x}{0.005 \text{ L}} = \dfrac{0.04 \text{ g}}{1 \text{ L}}$

40. $\dfrac{x}{24 \text{ hr}} = \dfrac{2 \text{ tablets}}{4 \text{ hr}}$

Identify the number of significant digits in the expressions below.

41. 0.0002
42. 0.1002
43. 5,000 mg
44. 5,001 g
45. 2 mg
46. 40 kg
47. 0.5 mg

Which of the following units are units of the same dimension?

48. ______ and ______

millimeters
cubic grams
cubic centimeters
centimeters

49. ____________ and ____________

grams
milliliters
kilometers
micrograms

50. ____________ and ____________

meters
milligrams
kiloliters
centimeters