Chapter 5

Word Problems

> "Then you should say what you mean," the March Hare went on.
>
> "I do," Alice hastily replied, "at least—at least I mean what I say—that's the same thing, you know."
>
> "Not the same thing a bit!" said the Hatter. "Why you might just as well say that 'I see what I eat' is the same thing as 'I eat what I see'!"
>
> —Lewis Carroll, *Alice's Adventures in Wonderland.*

Exhibit 5-1

Objectives

Upon completion of this chapter the clinician should be able to:

1. List three steps in setting up equations to solve dosage calculation problems from verbal orders
2. Set up word problems as algebraic equations
3. Solve algebraic equations, making necessary unit conversions to express the answer with the fewest non-integer factors
4. Given a concentration for a medicine, calculate the mass for any volume
5. Given a concentration for a medicine, calculate the volume for any mass

Key Terms

bolus
concentration
infusion
mass
mEq
milliequivalent
order of operations
sum
volume

WORD PROBLEMS

Physician orders for medication are often given verbally. This extends the physician's responsibility to the clinician. The clinician is ethically and legally responsible to administer the correct amount of medication. No health care provider should administer any medications or dose unless he is absolutely certain it is correct.

No health care provider should administer any dose until absolutely certain it is correct.

Exhibit 5-2 Clinician Responsibility

A clinician who is uncertain about a medication should advise the physician, check reference sources, and question the order. The clinician must confirm the following eight things: (1) the medication is right; (2) the dose is right; (3) the route is right; (4) the time for the dose is right; (5) it is being given to the right patient (confirm a written identification); (6) the patient has no medication allergies; (7) the patient consents to treatment; and (8) the clinician properly documents the medication administration. If the clinician's concern about any one of these is not resolved, he should advise the physician that he is unable to carry out the order and ask for an alternative. Some physicians may feel this challenges their authority, but most will appreciate the clinician's professionalism. Court cases have established the precedent that the clinician is personally responsible for medications he administers, even if complying with a physician's incorrect order.

Approach to Word Problems

The correct dose for some verbal orders is easily deduced intuitively, in the clinician's thoughts, without setting up an equation. This shortcut has risks. There is an increased risk of error when the clinician does not understand the process by which the answer is derived. Intuitive deduction can cause the clinician difficulty when he encounters more challenging problems. Without practice at setting up necessary equations, a clinician may be unable to solve the problem.

Setting up equations is simple if an organized process is used. There are three basic steps common to setting up an equation to solve any dosage calculation word problem:

1. Identify what is known. This includes what is spoken in an order. It also includes information available on the medication container. The

Figure 5-1 Nitroglycerine

medication container gives a concentration ratio of mass (usually grams or milligrams) to milliliters (or liters). Known information also includes basic knowledge, such as a working familiarity of the metric system.

2. Identify what is unknown and is asked for. A problem may have preliminary steps before the measured dose may be calculated (such as converting units of measure to like units). Many students have difficulty interpreting word problems, but once the *needed* information is identified, a ratio equation, using the known information, can be set up to solve for the unknown data. The clinician usually must know the metric dose or volume of the dose. One of those is usually given, and the other must be calculated.

3. Set up a ratio equation to solve for the unknown information. Many equations require a known concentration ratio. They also require either the volume or the weight of medication. An equation can only be solved for one unknown. Problems that require a solution for more than one unknown factor must have an equation for each.

1. Identify what is known.
2. Identify what is unknown.
3. Set up an equation and solve for the unknown.

Exhibit 5-3 Three Steps to Solving Word Problems

CONCENTRATIONS

There are several types of problems associated with verbal orders. These include calculation of infusion drip rates, mixing new concentrations, amount of medicine per unit of patient body weight, and rate of administration. The most common is concentration analysis.

Most verbal medication orders require calculating a volume of medication based on a mass. These are concentration problems. **Concentration** may be defined as an unchanging ratio of units of mass to units of volume.

Mass may be defined in this context as the weight of medicine. This is a bit simplified. The full scientific definition includes other considerations not necessary for dosage calculation.

Volume is the space occupied by a body. Most medications are fluid. A fluid volume of medication is most correctly expressed in dimensions of liters, such as milliliters (ml) or liters (L), rather than cubic area such as centimeters (cc).

In concentration problems, the concentration is provided on the medication label. A constant relationship exists between the units of volume and mass. The concentration may be used to determine any unit of mass for any given related unit of volume. Likewise, it may be used to determine any unit of volume for any given related unit of mass.

A concentration may be used to determine any unit of mass for any related unit of volume or any unit of volume for any related unit of mass.

Exhibit 5-4 Concentration

The form in which the concentration is used in an equation is not limited by how it is expressed. It may be used with either part (mass or volume) of the expression in the numerator.

The manner in which a concentration is set up in an equation is determined by the unknown. The easiest manner to set up an equation is to place the unknown in the numerator. Place the related known concentration (or volume) in the denominator. The given concentration is then set up on the other side of the equation in a parallel manner (if volume is on the bottom, the denominator) on the left, place volume on the bottom on the right), as shown in Exhibit 5-5. If *x*, the unknown, has milligrams in the denominator, then the concentration will be set up with milligrams in the denominator. If *x* has milliliters in the denominator, then the concentration will have milliliters in the denominator. See Exhibit 5-6 and Exhibit 5-7 for example concentration problems.

The unknown is set up in the numerator as *x*, and its related reference is placed in the denominator. The concentration is then set up in a parallel form.

$$\frac{x}{\text{ordered volume}} = \frac{\text{given mass}}{\text{given volume}}$$

There is parallelism between the denominators and numerators.

Exhibit 5-5 Setting Up Concentration Problems

Given a concentration of 100 mg/10 ml:

The physician order is to give 5 ml. How many milligrams are given?

First: The concentration is known to be 100 mg/10 ml.
The dose volume is known to be 5 ml.

Second: The unknown element is the metric dose.

Third: To set up the equation, we begin with the left side of the equation as *x* and then plug in the known data. We will place the metric dose of the concentration in the numerator because the metric dose of the order is what we are seeking.

$$\frac{x}{5\text{ ml}} = \frac{100\text{ mg}}{10\text{ ml}}$$

The last step is to solve for *x*, which is 50 mg.

Exhibit 5-6 Example Concentration Problem

The same problem in Exhibit 5-6 could be set up to ask for the dose volume instead of the metric dose. We merely set up the equation with volume in the numerator.

Given a concentration of 100 mg/10 ml:

The physician order is to give 50 mg. How many milliliters are given?

First:	The concentration is known to be 100 mg/10 ml. The metric dose is known to be 50 mg.
Second:	The unknown element is the dose volume.
Third:	To set up the equation, we begin with the left side of the equation as *x* and then plug in the known data. We will place the volume of the concentration in the numerator because the dose volume of the order is what we are seeking.

$$\frac{x}{50\text{ mg}} = \frac{10\text{ ml}}{100\text{ mg}}$$

The last step is to solve for *x*, which is 5 ml.

Exhibit 5-7 Example Concentration Problem

A three-part concentration such as:

factor:	100 mg /	10 ml /	prefilled syringe
	——	——	——
part:	1	2	3

will have only two of the three parts used at one time. Any two of the three may be used in a parallel set-up. Exhibit 5-8 solves first for the metric dose, as Exhibit 5-7 did, and then solves for the number of syringes. The point of this example is that any part of a concentration can be used to solve a question.

A solution measured in percent (%) indicates a ratio of medication per 100 ml of solution. Calcium chloride is an electrolyte. Electrolytes are measured in milliequivalents rather than grams or milliliters and is expressed in percent.

Presupplied electrolyte solutions can be another example of a multiple-part ratio. A medication is supplied in prefilled syringes of a 10% solution containing 1 g of medication in 10 ml. Electrolytic solutions are measured

Given a concentration of 100 mg/10 ml/prefilled syringe:

The physician order is for 5 ml. How many milligrams are given?

First:	The concentration is known to be 100 mg/10 ml/1 syringe.
Second:	We are asked for the metric dose in milligrams.
Third:	We set up an equation and place the metric dose in the numerator.

$\frac{X}{\rule{1cm}{0.4pt}} = \rule{1cm}{0.4pt}$ Start with our standard equation form.

$\frac{x}{5 \text{ ml}} = \frac{100 \text{ mg}}{10 \text{ ml}}$ We plug in the known data and solve for x, which is 50 mg.

The clinician needs to know how many prefilled syringes will be needed to give the dose. An equation can be used to find the number of syringes.

The same three steps are used. The second step now asks for syringes instead of the metric dose.

$x = \rule{1cm}{0.4pt}$ Start with our standard equation form.

$\frac{x}{50 \text{ mg}} = \frac{1 \text{ syringe}}{100 \text{ mg}}$ We plug in the known data and solve for x, which is 0.5 syringes.

Exhibit 5-8 Example Concentration Problem

in **milliequivalents (mEq)**. There are 13.6 mEq in a 10 ml syringe of 10% solution. The supply ratio for this medication has four parts, as shown in Exhibit 5-9. See Exhibit 5-10 for a sample problem using milliequivalents. Many electrolyte solutions are too concentrated for administration to infants, children, or patients with compromised circulation. A clinician must be prepared to dilute a premixed solution to a lower concentration to safely administer it.

Concentration	1 prefilled syringe/	1 g/	10 ml/	13.6 mEq
Part	1	2	3	4

Exhibit 5-9 Electrolyte Concentration

You are supplied a prefilled syringe with 1 g of an electrolytic medication in a 10 ml syringe of 10% solution containing 13.6 mEq (milliequivalents).

The physician orders you to give 20 mEq. How many milliliters do you administer, and how many prefilled syringes will you need to give it?

First:	Known data:	1 g/	10 ml/	10%/	13.6 mEq/	1 syringe
	Physician's order:				20 mEq	

This is a lot of data, so we need to organize it to see what we need to solve the problem and leave out what we don't need.

Second:	Needed data:	? ml	? mEq
Third:	$\frac{x}{\quad} = \frac{\quad}{\quad}$	Start with our standard equation.	

$$\frac{x}{20\text{ mEq}} = \frac{10\text{ ml}}{13.6\text{ mEq}}$$

Plug in the known data.
Solve for x.

$$(\cancel{20\text{ mEq}})\frac{x}{\cancel{20\text{ mEq}}} = \frac{10\text{ ml}}{13.6\ \cancel{\text{mEq}}}(20\ \cancel{\text{mEq}})$$

$$x = \frac{200\text{ ml}}{13.6}$$

Solve for x.

$x = 14.7$ ml — This gives us the volume of the dose.

To solve for the number of syringes, we use the same first and second steps and set up the equation for syringes instead of milliequivalents.

$$\frac{x}{20\text{ mEq}} = \frac{1\text{ syringe}}{13.6\text{ mEq}}$$

$x = 1.47$ syringes. The problem has only two significant figures, so the answer is rounded to 1.5 syringes, which is two significant figures.

Common sense tells us syringes are counted in whole numbers, so while we only give medication of 1.5 syringes, we will need 2 syringes, which is the closest whole number above 1.5.

Exhibit 5-10 Example Milliequivalent Problem

Any two parts of this "supply ratio" may be used in a parallel manner in an equation.

See Exhibit 5-11: an order is given for 75 mg of 2% lidocaine to be given as an IV bolus. The syringe is measured in milliliters, not milligrams.

The physician orders 75 mg of 2% lidocaine to be administered.
The medication is packaged in a syringe of 100 mg/5 ml.

First:	The concentration is known to be	100 mg/5 ml.
	The metric dose is known to be	75 mg.
Second:	The unknown element is the dose volume in milliliters.	
Third:	Set up an equation.	

$$\frac{x}{\quad} = \frac{\quad}{\quad}$$

$$\frac{x}{75\text{ mg}} = \frac{5\text{ ml}}{100\text{ mg}}$$

Milliliters are needed, so we set up the concentration side of the equation with milliliters in the numerator.

$$(\cancel{75\text{ mg}})\ \frac{x}{\cancel{75\text{ mg}}} = \frac{5\text{ ml}}{100\ \cancel{\text{mg}}}\ (75\ \cancel{\text{mg}})$$

$$x = \frac{375\text{ ml}}{100}$$

$$x = 3.75\text{ ml}$$

Exhibit 5-11 Example Concentration Problem

How much medication (in milliliters) is used from the syringe to give 75 mg?

See Exhibit 5-12. A physician orders medication in a dose of 5 mg/kg of patient weight to be given as an IV bolus. The medication is supplied in a prefilled syringe measured in milliliters. How much medication is used from the syringe to give 5 mg/kg? The patient weighs 132 lb. The medication label reads “500 mg/10 ml” in the prefilled syringe.

Exhibit 5-12 is a concentration problem. Given an order for medication based on patient weight, the clinician must calculate both the mass and the related volume for the dose. Solution of this problem requires more than one step because the related mass is not *directly* given. The related mass is expressed in milligrams per kilogram of patient weight. Patient weight is expressed in pounds.

There are three problems to be solved: first, the patient’s weight in kilograms; second, the related mass of medicine; and third, the related volume of the medicine to be given. The number of calculations adds complexity to the question, but a step-by-step approach using the equation set-up process simplifies it. Each calculation gives additional data used for the next step.

This problem involves three calculations. The order is based on patient weight in kilograms. The first calculation will be the patient's weight in kilograms. The second calculation will be the metric dose, and the third will be the dose volume. We can use *x*, *y*, and *z* for the unknown to distinguish between each of the calculations.

First:	Concentration: 500 mg/10 ml Order: 5 mg/kg patient weight Patient weight: 132 lb Unit conversion factor for kilograms: 2.204 lb/1 kg	
Second:	Needed:	patient weight in kilograms metric dose in milligrams dose volume in milliliters
Third:	$\frac{x}{\quad} = \frac{\quad}{\quad}$	

$\frac{x}{132 \text{ lb}} = \frac{1 \text{ kg}}{2.2 \text{ lb}}$	We set up a unit conversion factor to calculate patient weight. There are only two significant figures to this, so we can use 2.2 with adequate accuracy.
$x = 60$ kg	This is the patient weight in kilograms.
$\frac{y}{60 \text{ kg}} = \frac{5 \text{ mg}}{1 \text{ kg}}$	We set up the second calculation, which is for the metric dose. Milligrams are in the numerator because that is what is needed.
$y = 300$ mg	This is the metric dose.
$\frac{z}{300 \text{ mg}} = \frac{10 \text{ ml}}{500 \text{ mg}}$	We set up the third calculation, which is for dose volume. Milliliters are in the numerator. because that is what is needed.
$z = 6$ ml	This is the dose volume.

Exhibit 5-12 Example Concentration Problem

INFUSION AND BOLUS

Some of the terms used in verbal orders specify the manner in which a medication is to be administered. Intravenous therapy may be either by infusion or by bolus.

Infusion is a prolonged, controlled introduction of fluid into the body. This allows a very controlled rate of medication administration. Rates of infusion administration are commonly calculated in milliliters per hour. Intravenous (IV) infusions allow medications that may be too caustic or too concentrated for normal IV injection to be given intravenously in a diluted mixture. Infusion calculations are discussed in Chapter 8.

A mass of medication given as a single unit is a **bolus**. An IV bolus is a unit of medication injected as a single dose into a vein. The rate of the bolus injection depends on the medication being given. Intravenous boluses of small amounts should usually be given over 5 to 10 minutes. This provides the medication at a rate best suited to the body's ability to adsorb it. A rare number of medications must be given by rapid IV bolus (as quickly as possible) but most are administered during 5 to 10 minutes. A clinician must know the recommended routes and rates of each medication before administering it. It is usually the route of choice for administering medication in emergencies.

Total Daily Dose

A total daily dosage is the **sum** of all individual doses given in a 24-hour day. Occasionally, it may be necessary to determine an individual dose based on the total daily dosage and the number of times during the day the medication is given. Frequently used abbreviations to indicate the number of doses in a day include BID for twice a day, TID for three times a day, and QID for four times a day. A list of additional abbreviations is included in Appendix B.

Order of Operations

Mathematical **order of operations** refers to the sequence in which multiple mathematical operations should be performed if there is more than one type of operation in an equation. This was discussed in Chapter 2. A clinician should use parentheses to indicate the order of operations in equations that involve complex mathematical operations.

Review Problems

Given the concentration on the left, calculate the metric dose for the concentration per unit of volume.

1. 1 g/L ________ mg/ml ________ mg/500 ml
2. 2 g/L ________ mg/ml ________ g/500 ml
3. 1 mg/L ________ mcg/ml ________ mg/500 ml
4. 1 mcg/L ________ ng/ml ________ mcg/250 ml
5. 1 mg/0.5 L ________ mcg/ml ________ mcg/250 ml
6. 1 mg/0.25 L ________ mg/L ________ mcg/ml

7. 2 g/0.5 L	_______ g/250 ml	_______ mg/ml
8. 4 mg/ml	_______ g/L	_______ g/250 ml
9. 4 mcg/ml	_______ mg/L	_______ mg/250 ml
10. 4 mcg/ml	_______ mcg/250 ml	_______ mcg/L

Solve the following word problems using the partial equation provided. The unknown and its related factor are set up. Identify the proper ratio to use, complete the equation, and solve for *x*. Remember to use parallelism in the set-up.

11. How many grams would a clinician administer if a physician ordered 5 mg of morphine sulphate? It is supplied in a 15 mg/ml vial. _______
12. A physician ordered 0.10 g of lidocaine. How many milligrams will the clinician administer? It is supplied in 100 mg/10 ml prefilled syringe. _______
13. How many milliliters are in 0.25 L? _______
14. If a physician ordered 2.5 L of dextrose 5% in water, how many milliliters should be given? _______
15. The order is to give 2 tablets of acetylsalicylic acid. The tablets contain 350 mg of medication each. What is the metric dose given (in milligrams)? _______
16. A physician orders a bolus of lidocaine based on the ratio of 1 mg/kg of patient body weight. The patient weighs 110 lb. The lidocaine is supplied in 100 mg/10 ml syringe. How many milligrams does the clinician administer? _______
17. The label of a vial of medication reads "100 mg/10 ml." If the clinician is giving 1 mg/kg for a 110 lb patient, how many milliliters are administered? _______
18. The order is to give 50 mg. The label of the medication reads "1 g/10 ml." How many milliliters does the clinician administer? _______
19. One dose of a medication is 1 g. It is given TID. How many grams are given in a day? _______
20. A total day's dosage is 4 g. It is administered QID. How much medication is given with each dose? _______
21. A physician orders 10 mg/lb (of patient body weight). The patient weighs 45.45 kg. What is the dose? _______
22. A physician orders 5 mg of morphine sulphate, given BID. What is the total daily dose? _______
23. A physician orders 75 mg of meperidine for a patient. It is supplied in a 50 mg/ml vial. The vial contains 15 ml. How many milliliters does the clinician administer? _______

24. The doctor's orders read: "give 160 mg furosemide." It comes supplied in a container labeled "40 mg/ml." How many milliliters does the clinician administer? ______
25. If 8 ml contains 100 mg, how many milliliters contain 1,250 mg? ______

Use the following information to answer problems 26–28.

A 45.45 kg patient is prescribed 10 mg/lb of a medication to be given TID. The medication is marked "1 g/10 ml."

26. What is the dose? ______
27. How many milliliters are administered? ______
28. What is the total daily dose? ______

Solve problems 29–30 using the following information. Identify the proper ratio to use, set up an equation, and solve for x.

The doctor's order is for 1 mg of lidocaine per kilogram of patient body weight. The patient weighs 275 lb. The lidocaine is marked "2% solution, 100 mg/5 ml."

29. How many milligrams does the clinician administer? ______
30. How many milliliters does the clinician administer? ______

Solve problem 31 using the following information. Identify the proper ratio to use, set up an equation, and solve for x.

An order is given for 0.5 mg of atropine. It is supplied in a vial marked "1 g/30 ml."

31. How many milliliters does the clinician administer? ______

Solve problems 32–33 using the following information. Identify the proper ratio to use, set up an equation, and solve for x.

The doctor in question 31 decided to have the clinician use prefilled syringes, which contain 0.4 mg atropine each. Each syringe is 2 ml in volume.

32. How many prefilled syringes will the clinician use? ______
33. How many milliliters will the clinician administer? ______

Solve problems 34–35 using the following information. Identify the proper ratio to use, set up an equation, and solve for x.

A physician orders one half of a 10 ml prefilled syringe of calcium chloride to be administered. The prefilled syringe contains 1 g of 10% medication.

34. How much medication does the clinician administer? ______
35. How many milliliters does the clinician administer? ______

Solve problems 36–38 using the following information. Identify the proper ratio to use, set up an equation, and solve for x.

The doctor's orders are to administer 500 mg of a medication per mouth QID. The medication is supplied in 125 mg units.

36. How many units are given in each dose? _______
37. How many doses are given in a 24-hour day? _______
38. How many total units are given in a day? _______

Solve problems 39–40 using the following information. Identify the proper ratio to use, set up an equation, and solve for x.

A paramedic received an order to administer 1.5 mg/lb of patient body weight to a patient who weighs 90.9 kg. The medication is supplied in a vial containing 1 g in 10 ml.

39. How many milligrams does the clinician administer? _______
40. How many milliliters does the clinician administer? _______

Solve problems 41–43 using the following information. Identify the proper ratio to use, set up an equation, and solve for x.

A physician orders 20 ml of calcium chloride. Calcium chloride is supplied in prefilled syringes of 10% solution containing 10 ml and 1 g and 13.6 mEq.

41. How many milligrams does the patient receive? _______
42. How many syringes are used? _______
43. How many milliequivalents are given? _______

Solve problems 44–45 using the following information. Identify the proper ratio to use, set up an equation, and solve for x.

A patient weighs 185 lb. A physician orders a bolus of lidocaine based on the ratio of 1 mg/kg of body weight for this patient. The medication is supplied in a prefilled syringe of 100 mg/10 ml.

44. How many milligrams are given? _______
45. How many milliliters are given? _______

46. Define parallelism and how it applies to setting up dosage calculation equations.

____

____

____

47. Define concentration.

____

____

____

List the three steps to setting up an equation.

48. ____________________

49. ____________________

50. ____________________