Second Printing Errata for *Advanced Engineering Mathematics,* by Dennis G. Zill and Michael R. Cullen

(Yellow highlighting indicates corrected material)

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The second equation in (5) is the result of using partial fractions on the left side of the first equation. Integrating and using the laws of logarithms gives

$$\frac{1}{4}\ln|y-2| - \frac{1}{4}\ln|y+2| = x + c_1 \quad \text{or} \quad \ln\left|\frac{y-2}{y+2}\right| = 4x + c_2 \quad \text{or} \quad \frac{y-2}{y+2} = e^{4x+c_2}.$$

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Solution The equation is in standard form, and $P(x) = 1$ and $f(x) = x$ are continuous on $(-\infty, \infty)$. The integrating factor is $e^{\int dx} = e^x$, and so integrating

$$\frac{d}{dx}[e^x y] = xe^x$$

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19. $(4t^3y - 15t^2 - y)\,dt + (t^4 + 3y^2 - t)\,dy = 0$

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Example 5 Mixture of Two Salt Solutions

Recall that the large tank considered in Section 1.3 held 300 gallons of a brine solution. Salt was entering and leaving the tank; a brine solution was being pumped into the tank at the rate of 3 gal/min, mixed with the solution there, and then the mixture was pumped out at the rate of 3 gal/min. The concentration of the salt in the inflow, or solution entering, was 2 lb/gal, and so salt was entering the tank at the rate R_{in} = (2 lb/gal) · (3 gal/min) = 6 lb/min and leaving the tank at the rate R_{out} = (x/300 lb/gal) · (3 gal/min) = x/100 lb/min. From this data and (6) we get equation (8) of Section 1.3. Let us pose the question: If there were 50 lb of salt dissolved initially in the 300 gallons, how much salt is in the tank after a long time?

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13. **Leaking Conical Tank** A tank in the form of a right-circular cone standing on end, vertex down, is leaking water through a circular hole in its bottom.

    (a) Suppose the tank is 20 feet high and has radius 8 feet and the circular hole has radius 2 inches. In Problem 14 in Exercises 1.3 you were asked to show that the differential equation governing the height h of water leaking from a tank is

    $$\frac{dh}{dt} = -\frac{5}{6h^{3/2}}.$$

    In this model, friction and contraction of the water at the hole were taken into account with $c = 0.6$, and g was taken to be 32 ft/s^2. See Figure 1.30. If the tank is initially full, how long will it take the tank to empty?

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Basic Definition If $f(t)$ is defined for $t \geq 0$, then the improper integral $\int_0^\infty K(s, t)f(t)\,dt$ is defined as a limit:

$$\int_0^\infty K(s, t)f(t)\,dt = \lim_{b\to\infty}\int_0^b K(s, t)f(t)dt. \qquad (1)$$

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Transforming (19) with respect to the variable x gives

$$EI\Big(s^4Y(s) - s^3y(0) - s^2y'(0) - sy''(0) - y'''(0)\Big) = \frac{2w_0}{L}\left[\frac{L/2}{s} - \frac{1}{s^2} + \frac{1}{s^2}e^{-Ls/2}\right]$$

or

$$s^4Y(s) - sy''(0) - y'''(0) = \frac{2w_0}{EIL}\left[\frac{L/2}{s} - \frac{1}{s^2} + \frac{1}{s^2}e^{-Ls/2}\right].$$

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Similarly,

$$\mathscr{L}\{t^2f(t)\} = \mathscr{L}\{t\cdot tf(t)\} = -\frac{d}{ds}\mathscr{L}\{tf(t)\}$$

$$= -\frac{d}{ds}\left(-\frac{d}{ds}\mathscr{L}\{f(t)\}\right) = \frac{d^2}{ds^2}\mathscr{L}\{f(t)\}.$$

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Example 1 General Solution: ν Not an Integer

By identifying $\nu^2 = \frac{1}{4}$ and $\nu = \frac{1}{2}$ we can see from (9) that the general solution of the equation $x^2y'' + xy' + (x^2 - \frac{1}{4})y = 0$ on $(0, \infty)$ is $y = c_1J_{1/2}(x) + c_2J_{-1/2}(x)$. ❑

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Here the weights w_i, $i = 1, 2, \ldots, m$ are constants that generally satisfy $w_1 + w_2 + \cdots + w_m = 1$ and each k_i, $i = 1, 2, \ldots, m$ is the function f evaluated at a selected point (x, y) for which $x_n \leq x \leq x_{n+1}$. We shall see that the k_i are defined recursively. The number m is called the **order** of the method. Observe that by taking $m = 1$, $w_1 = 1$ and $k_1 = f(x_n, y_n)$ we get the familiar Euler formula $y_{n+1} = y_n + hf(x_n, y_n)$. Hence Euler's method is said to be a **first-order Runge-Kutta method**.

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THEOREM 7.2 Criterion for Parallel Vectors

Two nonzero vectors **a** and **b** are **parallel** if and only if $\mathbf{a} \times \mathbf{b} = \mathbf{0}$.

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Solution The vectors $\overrightarrow{P_1P_2}$ and $\overrightarrow{P_2P_3}$ can be taken as two sides of the triangle. Since $\overrightarrow{P_1P_2} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{P_2P_3} = \mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$, we have

$$\overrightarrow{P_1P_2} \times \overrightarrow{P_2P_3} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 1 & -3 & -5 \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ -3 & -5 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 1 & 3 \\ 1 & -5 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 1 & 2 \\ 1 & -3 \end{vmatrix}\mathbf{k}$$

$$= -\mathbf{i} + 8\mathbf{j} - 5\mathbf{k}.$$

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DEFINITION 8.5 Scalar Multiple of a Matrix

If k is a real number, then the **scalar multiple** of a matrix $\mathbf{A}$ is

$$k\mathbf{A} = \begin{pmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & & & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{pmatrix} = (ka_{ij})_{m \times n}.$$

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Example 1 Matrix with Two Identical Rows

Since the second and third columns in the matrix $\mathbf{A} = \begin{pmatrix} 6 & 2 & 2 \\ 4 & 2 & 2 \\ 9 & 2 & 2 \end{pmatrix}$ are the same, it follows from Theorem 8.9 that

$$\det \mathbf{A} = \begin{vmatrix} 6 & 2 & 2 \\ 4 & 2 & 2 \\ 9 & 2 & 2 \end{vmatrix} = 0.$$

You should verify this by expanding the determinant by cofactors. ❑

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Multiplying these vectors, in turn, by the reciprocals of the norms $\|\mathbf{K}_1\| = \sqrt{3}$, $\|\mathbf{K}_2\| = \sqrt{6}$, and $\|\mathbf{K}_3\| = \sqrt{2}$, we obtain an orthonormal set

$$\begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}, \begin{pmatrix} -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{pmatrix}, \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}.$$

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Subtracting (6) from (5) and using the fact that $\partial^2 f/\partial x \partial y = \partial^2 f/\partial y \partial x,$ we see that (4) becomes, after rearranging,

$$\iint_R \left[-\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\frac{\partial f}{\partial x} - \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\frac{\partial f}{\partial y} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\right] dA.$$

This last expression is the same as the right side of (3), which was to be shown. ❑

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Therefore,
$$\iint_S (\text{curl } \mathbf{F} \cdot \mathbf{n})\, dS = \iint_S \frac{-2xy - x}{\sqrt{4x^2 + 1}}\, dS.$$

To evaluate the latter surface integral, we use (5) of Section 9.13:

$$\begin{aligned} \iint_S \frac{-2xy - x}{\sqrt{4x^2 + 1}}\, dS &= \iint_R (-2xy - x)\, dA \\ &= \int_0^1 \int_{-2}^{2} (-2xy - x)\, dy\, dx \\ &= \int_0^1 \Big[-xy^2 - xy \Big]_{-2}^{2} dx \\ &= \int_0^1 (-4x)\, dx = -2. \end{aligned} \tag{7}$$

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■ **Triple Integrals in Cylindrical Coordinates** Recall from Section 9.11 that the area of a **polar rectangle** is $\Delta A = r^* \Delta r\, \Delta\theta$, where r^* is the average radius. From Figure 9.129(a) we see that the volume of a **cylindrical wedge** is simply $\Delta V =$ (area of base)(height) $= r^* \Delta r\, \Delta\theta\, \Delta z$. Thus, if $F(r, \theta, z)$ is a continuous function over the region D, as shown in Figure 9.129(b), then the triple integral of F over D is given by

$$\iiint_D F(r, \theta, z)\, dV = \iint_R \left[\int_{f_1(r,\theta)}^{f_2(r,\theta)} F(r, \theta, z)\, dz \right] dA = \int_\alpha^\beta \int_{g_1(\theta)}^{g_2(\theta)} \int_{f_1(r,\theta)}^{f_2(r,\theta)} F(r, \theta, z)\, r\, dz\, dr\, d\theta.$$

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■ **Variation of Parameters** Analogous to the procedure in Section 3.5, we ask whether it is possible to replace the matrix of constants $\mathbf{C}$ in (2) by a column matrix of functions

$$\mathbf{U}(t) = \begin{pmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_n(t) \end{pmatrix} \quad \text{so that} \quad \mathbf{X}_p = \mathbf{\Phi}(t)\mathbf{U}(t) \tag{4}$$

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Finally, when $c = -9$, $\lambda = -1 \pm \sqrt{-9} = -1 \pm 3i$. Thus the eigenvalues are conjugate complex numbers with negative real part -1. Figure 11.8(d) shows that solution curves spiral in toward the origin **0** as t increases. ❑

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Solution If we let $dx/dt = y$, then $dy/dt = x^3 - x$. From this we obtain the first-order differential equation

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{x^3 - x}{y},$$

which can be solved by separation of variables. Integrating

$$\int y\,dy = \int (x^3 - x)\,dx \quad \text{gives} \quad \frac{y^2}{2} = \frac{x^4}{4} - \frac{x^2}{2} + c.$$

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By orthogonality we have

$$\int_{-p}^{p} \cos\frac{m\pi}{p}x\,dx = 0, \quad m > 0, \qquad \int_{-p}^{p} \cos\frac{m\pi}{p}x \sin\frac{n\pi}{p}x\,dx = 0$$

and

$$\int_{-p}^{p} \cos\frac{m\pi}{p}x \cos\frac{n\pi}{p}x\,dx = \begin{cases} 0, & m \neq n \\ p, & m = n. \end{cases}$$

Finally, if we multiply (2) by $\sin(m\pi x/p)$, integrate, and make use of the results

$$\int_{-p}^{p} \sin\frac{m\pi}{p}x\,dx = 0, \quad m > 0, \qquad \int_{-p}^{p} \sin\frac{m\pi}{p}x \sin\frac{n\pi}{p}x\,dx = 0$$

and

$$\int_{-p}^{p} \sin\frac{m\pi}{p}x \sin\frac{n\pi}{p}x\,dx = \begin{cases} 0, & m \neq n \\ p, & m = n, \end{cases}$$

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Solution Here $T = 1 = 2p$ so $p = \frac{1}{2}$. Since f is 0 on the intervals $(-\frac{1}{2}, -\frac{1}{4})$ and $(\frac{1}{4}, \frac{1}{2})$, (8) becomes

$$\begin{aligned} c_n &= \int_{-1/2}^{1/2} f(x)e^{2in\pi x}\,dx = \int_{-1/4}^{1/4} 1 \cdot e^{2in\pi x}\,dx \\ &= \left.\frac{e^{2in\pi x}}{2in\pi}\right|_{-1/4}^{1/4} \\ &= \frac{1}{n\pi}\,\frac{e^{in\pi/2} - e^{-in\pi/2}}{2i}. \end{aligned}$$

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In view of (6) and (7) of Section 3.8, the product solutions (4) can be written as

$$u_n(x, t) = C_n \sin\left(\frac{n\pi a}{L} t + \phi_n\right) \sin \frac{n\pi}{L} x, \tag{11}$$

where $C_n = \sqrt{A_n^2 + B_n^2}$ and ϕ_n is defined by $\sin \phi_n = A_n/C_n$ and $\cos \phi_n = B_n/C_n$. For $n = 1, 2, 3, \ldots$ the standing waves are essentially the graphs of $\sin(n\pi x/L)$, with a time-varying amplitude given by

$$C_n \sin\left(\frac{n\pi a}{L} t + \phi_n\right).$$

Alternatively, we see from (11) that at a fixed value of x each product function $u_n(x, t)$ represents simple harmonic motion with amplitude $C_n|\sin(n\pi x/L)|$ and frequency $f_n = na/2L$. In other words, each point on a standing wave vibrates with a different amplitude but with the same frequency. When $n = 1$,

$$u_1(x, t) = C_1 \sin\left(\frac{\pi a}{L} t + \phi_1\right) \sin \frac{\pi}{L} x$$

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Example 2 Using the Cosine Transform

The steady-state temperature in a semi-infinite plate is determined from

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \qquad 0 < x < \pi, \quad y > 0$$

$$u(0, y) = 0, \qquad u(\pi, y) = e^{-y}, \quad y > 0$$

$$\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0, \qquad 0 < x < \pi.$$

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Hence from (7), the Fourier coefficients are given by $\mathbf{c} = \frac{1}{4}\overline{\mathbf{F}}_4\mathbf{f}$:

$$\begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{pmatrix}\begin{pmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \end{pmatrix}.$$

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Solution In this case, $r = \sqrt{2}$ and $\theta = \arg z = \pi/4$. From (10) with $n = 4$, we obtain

$$w_k = (\sqrt{2})^{1/4}\left[\cos\left(\frac{\pi/4 + 2k\pi}{4}\right) + i\sin\left(\frac{\pi/4 + 2k\pi}{4}\right)\right], \quad k = 0, 1, 2, 3.$$

Thus,

$$k = 0, \quad w_0 = (\sqrt{2})^{1/4}\left[\cos\frac{\pi}{16} + i\sin\frac{\pi}{16}\right] = 1.0696 + 0.2127i$$

$$k = 1, \quad w_1 = (\sqrt{2})^{1/4}\left[\cos\frac{9\pi}{16} + i\sin\frac{9\pi}{16}\right] = -0.2127 + 1.0696i$$

$$k = 2, \quad w_2 = (\sqrt{2})^{1/4}\left[\cos\frac{17\pi}{16} + i\sin\frac{17\pi}{16}\right] = -1.0696 - 0.2127i$$

$$k = 3, \quad w_3 = (\sqrt{2})^{1/4}\left[\cos\frac{25\pi}{16} + i\sin\frac{25\pi}{16}\right] = 0.2127 - 1.0696i.$$

❑

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Example 4 Order of Poles

(a) Inspection of the rational function

$$F(z) = \frac{2z + 5}{(z - 1)(z + 5)(z - 2)^4}$$

shows that the denominator has zeros of order 1 at $z = 1$ and $z = -5$, and a zero of order 4 at $z = 2$. Since the numerator is not zero at these points, it follows from Theorem 19.11 that F has simple poles at $z = 1$ and $z = -5$, and a pole of order 4 at $z = 2$.