First Printing Errata for *Advanced Engineering Mathematics,* by Dennis G. Zill and Michael R. Cullen

(Yellow highlighting indicates corrected material)

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I would like to thank the following individuals for their feedback and suggestions for improvement over the previous editions and of the preliminary versions of the new edition:

Sonia Henckel, Lawrence Technological University

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4. Show that $U(P) = C\pi R^2 \dfrac{2J_1(kRw)}{kRw}$. Therefore the intensity is given by

$$|U(P)|^2 = \left[\frac{2J_1(kRw)}{kRw}\right]^2 I_0$$

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(b) From the derivatives $y' = xe^x + e^x$ and $y'' = xe^x + 2e^x$ we have for every real number x,

left-hand side: $y'' - 2y' + y = (xe^x + 2e^x) - 2(xe^x + e^x) + xe^x = 0$
right-hand side: 0. ❑

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$$\begin{aligned}\frac{dx}{dt} &= f(t, x, y)\\ \frac{dy}{dt} &= g(t, x, y).\end{aligned} \qquad (9)$$

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If possible, find the text *Differential Equations*, Ralph Palmer Agnew, McGraw-Hill Book Co., and then discuss the construction and solution of the mathematical model.

38. Reread this section and classify each mathematical model as linear or nonlinear.

39. **Population Dynamics** Suppose that $P'(t) = 0.15\,P(t)$ represents a mathematical model for the growth of a certain cell culture, where $P(t)$ is the size of the culture (measured in millions of cells) at time t (measured in hours). How fast is the culture growing at the time t when the size of the culture reaches 2 million cells?

40. **Radioactive Decay** Suppose that

$$A'(t) = -0.0004332A(t)$$

represents a mathematical model for the decay of radium-226, where $A(t)$ is the amount of radium (measured in grams) remaining at time t (measured in years). How much of the radium sample remains at time t when the sample is decaying at a rate of 0.002 grams per year?

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Chapter Outline

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- If $(M_y - N_x)/N$ is a function of x alone, then an integrating factor for equation (11) is

$$\mu(x) = e^{\int \frac{M_y - N_x}{N} dx}. \quad (13)$$

- If $(N_x - M_y)/M$ is a function of y alone, then an integrating factor for equation (11) is

$$\mu(y) = e^{\int \frac{N_x - M_y}{M} dy}. \quad (14)$$

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7. Figure 2.64 is a portion of the direction field of a differential equation $dy/dx = f(x, y)$. By hand, sketch two different solution curves, one that is tangent to the lineal element shown in black and the other tangent to the lineal element shown in color.

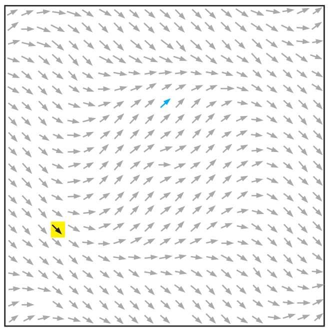

Figure 2.64 Direction field for Problem 7

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auxiliary equation with real coefficients, then its conjugate $m_2 = \alpha - i\beta$ is also a root of multiplicity k. From the $2k$ complex-valued solutions

$$e^{(\alpha+i\beta)x}, \quad xe^{(\alpha+i\beta)x}, \quad x^2e^{(\alpha+i\beta)x}, \quad \ldots, \quad x^{k-1}e^{(\alpha+i\beta)x}$$
$$e^{(\alpha-i\beta)x}, \quad xe^{(\alpha-i\beta)x}, \quad x^2e^{(\alpha-i\beta)x}, \quad \ldots, \quad x^{k-1}e^{(\alpha-i\beta)x}$$

The quadratic formula then yields the remaining roots $m_2 = -1 + \sqrt{3}i$ and $m_3 = -1 - \sqrt{3}i$. Therefore the general solution of $3y''' + 5y'' + 10y' - 4y = 0$ is $y = c_1e^{x/3} + e^{-x}(c_2 \cos \sqrt{3}\,x + c_3 \sin \sqrt{3}x)$.

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7. A cantilever beam of length L is embedded at its right end, and a horizontal tensile force of P pounds is applied to its free left end. When the origin is taken at its free

15. $y'' + 2y' + (\lambda + 1)y = 0, \;\; y(0) = 0, \;\; y(5) = 0$

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In other words, the motion of the coupled system is represented by the system of linear second-order equations

$$\begin{aligned} m_1x_1'' &= -k_1x_1 + k_2(x_2 - x_1) \\ m_2x_2'' &= -k_2(x_2 - x_1). \end{aligned} \tag{1}$$

If an nth-order differential operator $a_nD^n + a_{n-1}D^{n-1} + \cdots + a_1D + a_0$ factors into differential operators of lower order, then the factors commute. Now, for example, to rewrite the system

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The stipulated initial conditions then imply $c_1 = 0$, $c_2 = -\sqrt{2}/10$, $c_3 = 0$, $c_4 = \sqrt{3}/5$. And so the solution of the initial-value problem is

$$\begin{aligned} x_1(t) &= -\frac{\sqrt{2}}{10}\sin\sqrt{2}t + \frac{\sqrt{3}}{5}\sin 2\sqrt{3}t \\ x_2(t) &= -\frac{\sqrt{2}}{5}\sin\sqrt{2}t - \frac{\sqrt{3}}{10}\sin 2\sqrt{3}t. \end{aligned} \tag{14}$$

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Convolution If functions f and g are piecewise continuous on $[0, \infty)$, then a special product, denoted by $f * g$, is defined by the integral

$$f * g = \int_0^t f(\tau)g(t-\tau)\,d\tau \tag{2}$$

and is called the **convolution** of f and g. The convolution $f * g$ is a function of t. For example,

$$e^t * \sin t = \int_0^t e^\tau \sin(t-\tau)\,d\tau = \frac{1}{2}(-\sin t - \cos t + e^t). \tag{3}$$

It can be shown that $\int_0^t f(\tau)g(t-\tau)\,d\tau = \int_0^t f(t-\tau)g(\tau)\,d\tau$, that is, $f * g = g * f$. This means that the convolution of two functions is commutative.

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The inverse form of (7),

$$\int_0^t f(\tau)\,d\tau = \mathscr{L}^{-1}\left\{\frac{F(s)}{s}\right\}, \tag{8}$$

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Solution From $xP(x) = 0$ and $x^2Q(x) = x$ and the fact that 0 and x are their own power series centered at 0, we conclude $a_0 = 0$ and $b_0 = 0$ and so from (14) the indicial equation is $r(r-1) = 0$. You should verify that the two recurrence relations corresponding to the indicial roots $r_1 = 1$ and $r_2 = 0$ yield exactly the same set of coefficients. In other words, in this case the method of Frobenius produces only a single series solution

$$y_1(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(n+1)!} x^{n+1} = x - \frac{1}{2}x^2 + \frac{1}{12}x^3 - \frac{1}{144}x^4 + \cdots. \quad \square$$

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Solution From the known solution given in Example 4,

$$y_1(x) = x - \frac{1}{2}x^2 + \frac{1}{12}x^3 - \frac{1}{144}x^4 + \cdots,$$

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$$y_2(x) = y_1(x)\int \frac{e^{-\int 0\,dx}}{[y_1(x)]^2}\,dx = y_1(x)\int \frac{dx}{\left[x - \frac{1}{2}x^2 + \frac{1}{12}x^3 - \frac{1}{144}x^4 + \cdots\right]^2}$$

$$= y_1(x)\int \frac{dx}{\left[x^2 - x^3 + \frac{5}{12}x^4 - \frac{7}{72}x^5 + \cdots\right]} \quad \leftarrow \text{after squaring}$$

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Direction Cosines For a nonzero vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ in 3-space, the angles α, β, and γ between $\mathbf{a}$ and the unit vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$, respectively, are called **direction angles** of $\mathbf{a}$. See Figure 7.34. Now, by (6),

$$\cos\alpha = \frac{\mathbf{a}\cdot\mathbf{i}}{\|\mathbf{a}\|\|\mathbf{i}\|}, \quad \cos\beta = \frac{\mathbf{a}\cdot\mathbf{j}}{\|\mathbf{a}\|\|\mathbf{j}\|}, \quad \cos\gamma = \frac{\mathbf{a}\cdot\mathbf{k}}{\|\mathbf{a}\|\|\mathbf{k}\|},$$

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Remarks

(*i*) Suppose V is an arbitrary real vector space. If there is an inner product defined on V it need not look at all like the standard or Euclidean inner product defined on R^n. In Chapter 12 we will work with an inner product that is a definite integral. We shall denote an inner product that is *not* the Euclidean inner product by the symbol $(\mathbf{u}, \mathbf{v})$. See Problems 30, 31, and 38(b) in Exercises 7.6.

(*ii*) A vector space V on which an inner product has been defined is called an **inner product space**. A vector space V can have more than one inner product defined on it. For example, a non-Euclidean inner product defined on R^2 is $(\mathbf{u}, \mathbf{v}) = u_1v_1 + 4u_2v_2$, where $\mathbf{u} = \langle u_1, u_2\rangle$ and $\mathbf{v} = \langle v_1, v_2\rangle$. See Problems 37 and 38(a) in Exercises 7.6.

(*iii*) A lot of our work in the later chapters in this text takes place in an infinite dimensional vector space. As such, we need to extend the definition of linear independence of a finte set of vectors $S = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n\}$ given in Definition 7.7 to an infinite set:

> An infinite set of vectors $S = \{\mathbf{x}_1, \mathbf{x}_2, \ldots\}$ is said to be **linearly independent** if *every* finite subset of the set S is linearly independent. If the set S is not linearly independent, then it is **linearly dependent**.

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DEFINITION 9.11 Surface Area

Let f be a function for which the first partial derivatives f_x and f_y are continuous on a closed region R. Then the **area of the surface over R** is given by

$$A(S) = \iint_R \sqrt{1 + [f_x(x, y)]^2 + [f_y(x, y)]^2}\, dA. \qquad (2)$$

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Computer Lab Assignments

In Problems 15 and 16, use a CAS or linear algebra software as an aid in finding the general solution of the given system.

Computer Lab Assignments

32. Find phase portraits for the systems in Problems 20 and 21. For each system, find any half-line trajectories and include these lines in your phase portrait.

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The entries in $\mathbf{X}_1$ form the first column of $\mathbf{\Phi}(t)$, and the entries in $\mathbf{X}_2$ form the second column of $\mathbf{\Phi}(t)$. Hence

$$\mathbf{\Phi}(t) = \begin{pmatrix} e^{-2t} & e^{-5t} \\ e^{-2t} & -2e^{-5t} \end{pmatrix} \quad \text{and} \quad \mathbf{\Phi}^{-1}(t) = \begin{pmatrix} \frac{2}{3}e^{2t} & \frac{1}{3}e^{2t} \\ \frac{1}{3}e^{5t} & -\frac{1}{3}e^{5t} \end{pmatrix}.$$

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Subject to: $A_1y(a) + B_1y'(a) = 0$ (4)

$A_2y(b) + B_2y'(b) = 0$ (5)

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Chapter Outline

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Boundary-Value Problems Problems such as

Solve: $a^2\dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial^2 u}{\partial t^2}, \quad 0 < x < L, \quad t > 0$

Subject to: (BC) $u(0, t) = 0, \quad u(L, t) = 0, \quad t > 0$ (11)

(IC) $u(x, 0) = f(x), \left.\dfrac{\partial u}{\partial t}\right|_{t=0} = g(x), \; 0 < x < L$

and

Solve: $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < a, \quad 0 < y < b$

Subject to: (BC) $\begin{cases} \left.\dfrac{\partial u}{\partial x}\right|_{x=0} = 0, & \left.\dfrac{\partial u}{\partial x}\right|_{x=a} = 0, & 0 < y < b \\ u(x, 0) = 0, & u(x, b) = f(x), & 0 < x < a \end{cases}$ (12)

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Discussion Problems

21. Solve the Neumann problem for a rectangle:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < a, \quad 0 < y < b$$

$$\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0, \quad \left.\frac{\partial u}{\partial y}\right|_{y=b} = 0, \quad 0 < x < a$$

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where A is constant. Solve this partial differential equation subject to

$$u(0, t) = 0,\ u(1, t) = 0,\ t > 0$$

$$u(x, 0) = 0,\ \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0,\ 0 < x < 1.$$

[repeated text removed]

10. A string initially at rest on the x-axis is secured on the x-axis at $x = 0$ and $x = 1$. If the string is allowed to fall

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14.2 Problems in Cylindrical Coordinates

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Solution The boundary conditions suggest that the temperature u has radial symmetry. Accordingly, $u\,(r, z)$ is determined from

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} = 0, \quad 0 < r < 2, 0 < z < 4$$

$$u(2, z) = 0, \quad 0 < z < 4$$

$$u(r, 0) = 0, \quad u(r, 4) = u_0, \quad 0 < r < 2.$$

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5. The temperature in a circular plate of radius c is determined from the boundary-value problem

$$k\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) = \frac{\partial u}{\partial t}, \quad 0 < r < c, t > 0$$

$$u(c, t) = 0, \quad t > 0$$

$$u(r, 0) = f\,(r), \quad 0 < r < c.$$

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14.3 Problems in Spherical Coordinates

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15. Find the steady-state temperature $u(r, \theta)$ in a sphere of unit radius if the surface is kept at

$$u(1, \theta) = \begin{cases} 100, & 0 < \theta < \pi/2 \\ -100, & \pi/2 < \theta < \pi. \end{cases}$$

[*Hint:* See Problem 22 in Exercises 12.6.]

16. Solve the boundary-value problem

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} = \frac{\partial^2 u}{\partial t^2}, \; 0 < r < 1, t > 0$$

$$\left.\frac{\partial u}{\partial r}\right|_{r=1} = 0, \; t > 0$$

$$u(r, 0) = f(r), \left.\frac{\partial u}{\partial t}\right|_{t=0} = g(r), 0 < r < 1.$$

17. The function $u(x) = Y_0(\alpha a)J_0(\alpha x) - J_0(\alpha a)Y_0(\alpha x)$, $a > 0$ is a solution of the parametric Bessel equation

$$x^2\frac{d^2u}{dx^2} + x\frac{du}{dx} + \alpha^2 x^2 u = 0$$

on the interval $a \leq x \leq b$. If the eigenvalues $\lambda_n = \alpha_n^2$ are defined by the positive roots of the equation

$$Y_0(\alpha a)J_0(\alpha b) - J_0(\alpha a)Y_0(\alpha b) = 0,$$

show that the functions

$$u_m(x) = Y_0(\alpha_m a)J_0(\alpha_m x) - J_0(\alpha_m a)Y_0(\alpha_m x)$$
$$u_n(x) = Y_0(\alpha_n a)J_0(\alpha_n x) - J_0(\alpha_n a)Y_0(\alpha_n x)$$

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Since the curve C_1 is defined by $y = x$, it makes sense to use x as a parameter. Therefore, $z(x) = x + ix$, $z'(x) = 1 + i$, $f(z(x)) = x^2 + ix^2$, and

$$\int_{C_1} (x^2 + iy^2)\,dz = \int_0^1 (x^2 + ix^2)(1 + i)\,dx$$
$$= (1 + i)^2\int_0^1 x^2 dx = \frac{(1 + i)^2}{3} = \frac{2}{3}i.$$

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THEOREM 18.3 A Bounding Theorem

If f is continuous on a smooth curve C and if $|f(z)| \leq M$ for all z on C, then $\left|\int_C f(z)\,dz\right| \leq ML$, where L is the length of C.