Using Geometer's SketchPad With Euclidean and Transformational Geometry By Shlomo Libeskind

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This document assumes the reader has some familiarity with the Geometer's SketchPad software. The reader who has never used Geometer's SketchPad should first look at a SketchPad book such as *Geometry in Action* by Clark Kimberling (Published by Key Curriculum Press). Reading Chapter 0 (Basics) and Chapter 1 is enough to start the fun with Sketchpad.

SketchPad can be used to develop geometric intuition and insight in students. Also, most students enjoy creating highly accurate drawings that can be easily manipulated and which can be constructed with beautiful colors and shadings. Geometer's SketchPad can also be used to mimic classical compass and straight edge constructions. The best way to use SketchPad with a class is to use a computer lab with equipment to project the teacher's computer screen so that all the students can see it. Alternately, the teacher can Prepare a worksheet that gives directions for a Sketchpad figure.

Assuming some in the class are not familiar with SketchPad, a good place to start is with the teacher demonstrating and the students experimenting with the tools in the toolbox on the left margin of the SketchPad screen. This might also be good time to show them the automatic point labeling feature that can be turned on using the **Preferences** dialog box on the **Edit** menu. Other possibilities are *Hide object* and *Undo.* Often there will be students who are familiar with SketchPad from other classes or from high school. These people can be partnered with SketchPad novices in the class.

Lab Exercise #1 *(Theorem 1.3)*

After sufficient time for experimenting, the class should be ready to tackle something from the text. While the examples in the Prologue lend themselves to SketchPad experiments, we recommend that beginners start with problems that use fewer SketchPad features in one sketch. The first problem we would like to do is the ***Theorem 1.3.*** The steps we used have been saved in this sketch (and every sketch) in a "*tool*" script that is embedded in each file. To view the script, first click on the *tool button* in the toolbox (last button, double arrow) then click on *Show Script.* Finally, click the same button again and click the name of the script (last item, the only script in this file).

For this sketch is best to set the precision of measurement (*Preferences:* **Edit Menu**) to

1. Whole degrees for angles (units)
2. Tenths for lengths

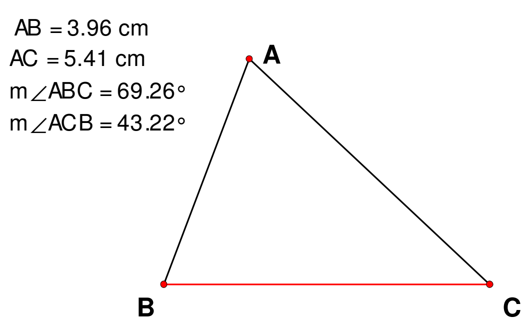
This sketch investigates the properties of isosceles triangles and related inequalities. Most students will remember the isosceles triangle theorem so it's not a good result for student discovery, but the related inequalities are less well known. That's why we like to have them construct any arbitrary triangle that is not necessarily isosceles.

This is also a good time to turn on automatic labeling (see the *Text* tab in *Preferences* on **Edit Menu).**

Now to begin the sketch:

1. Use the point tool from the toolbox to plot the three vertices of the triangle.
2. Highlight all three points and use *Segments* on the **Construct Menu** to construct the sides of the triangle. Let's assume that *BC* is the horizontal base at the bottom of the sketch and that *A* is above the base. To facilitate discussion highlight (only) the base and change it's color using *Color* on the **Display Menu.**
3. Highlight one of the vertical (non-base) sides and use *Length* on the **Measure Menu** to display its length. *Common Error Alert:* At this point someone may say "I did it and nothing happened." The most common error is to have the wrong things highlighted. If more than one side or a vertex is highlighted the command won't work. In general, you have to have exactly the correct things highlighted for a command to work. Then measure the other vertical side.
4. Next measure the base angles. To measure an angle you highlight the vertex and a point on each side. *Common Error Alert:* Order matters!! The computer assumes that the second of the three is the vertex. Then use *angle* on the **Measure Menu**. Then measure the other base angle.
5. Finally, show them how to drag a vertex to change the shape of the triangle and challenge them to make the vertical sides congruent to demonstrate the familiar theorem. Then see if they can discover what can be concluded about the base angles when the sides are not congruent. (What we're hoping for is: *The larger angle is opposite the larger side*).

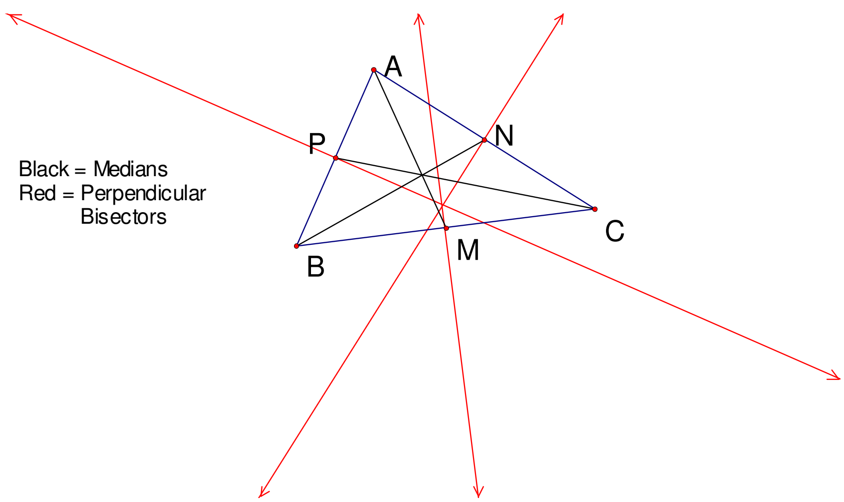
The completed sketch should look as follows.



Lab Exercise #2 ***(Now Solve This 1.3)***

We recommend creating two separate sketches, with six of the constructed segments or lines in each and putting all 12 together in one sketch later.

The assignment is to draw, for an arbitrary scalene triangle, 1)the medians, 2)the perpendicular bisectors of the sides, 3)the angle bisectors and 4)the altitudes. First, construct a triangle with vertices **A, B** and **C.**
We will use this triangle for the medians and perpendicular bisectors as shown here.



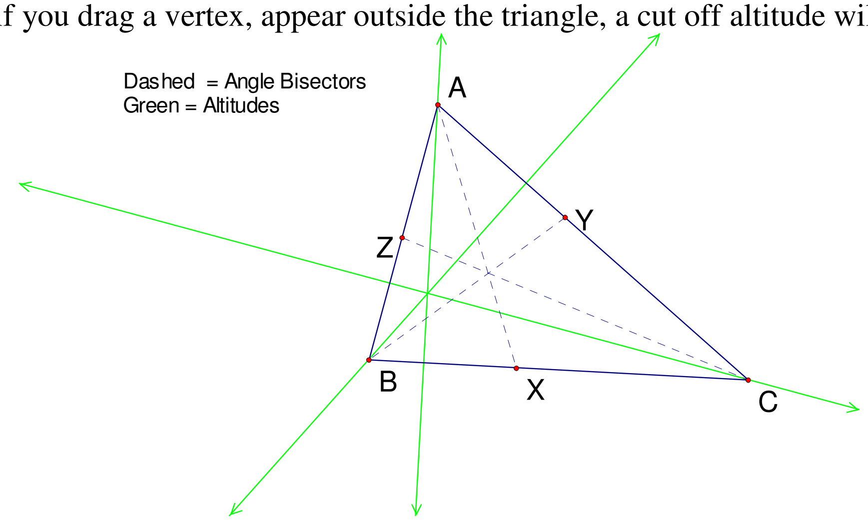
1) Medians
   a) Remind the class to turn on automatic labeling
   b) Use the *Point Tool* in the *Toolbox* to plot three points.
   c) Highlight the points and construct *Segments* (**Construct Menu)**
   d) Highlight the three segments and construct *Midpoints* (**Construct Menu).**
      Common Error Alert: A common occurrence is nothing happens. The cause is usually that some of the vertices are also highlighted. Remind the students they must have exactly the correct things highlighted for a command to work.
   e) Highlight a vertex and the opposite midpoint and construct the median (*Construct Segment*: **Construct Menu)**. Do this for the other two vertex-median pairs.
      Common Error Alert: The midpoints in the preceding step can be constructed all at once. The medians must be done one at a time.
   f) Show how to vary the triangle by dragging a vertex and see if the class can guess the concurrency theorem.

2) The Perpendicular Bisectors
   a) Highlight a midpoint (already constructed) and its side. Construct *Perpendicular* (**Construct Menu).** With the same procedure, construct the other two perpendicular bisectors. This is a good place to demonstrate the color feature of SketchPad. Highlight the three segments just constructed and, from the **Display Menu,** select *Color* and then select a color you like.

The other construction asked for are the angle bisectors and the altitudes. We recommend opening a new sketch (**File Menu**).

3) Angle Bisectors
   As above, using the toolbox, plot three vertices and then, using the **Construct Menu,** construct the sides (*Segments)* of the triangle.
   a) Next, one at a time, construct the angle bisectors using the **Construct Menu.** For each one you must highlight the vertex and a point on each ray of the angle. <u>*Common Error Alert:*</u> The vertex *must* be the second thing you highlight. For example, to construct the bisector at vertex **A**, highlight **B, A, C** in that order then select *Angle Bisector* on the **Construct Menu.** The vertices can be selected in the order **C, A, B** as well.
   b) We like to "cut off" the part of each bisector that is outside the triangle. To do this select a bisector and the side it intersects and select *Intersection* from the **Construct Menu**. Next select <u>only</u> the bisector and hide it using *Hide* on the **Display Menu.** Finally, highlight the point just constructed and the vertex of the angle and construct that segment.

4) Altitudes (constructed one at a time):
   Select a vertex and the opposite side and then select *Perpendicular Line* from **Construct Menu**. Then do the same for the other vertex-side pairs. This is also a good place to change colors to distinguish the altitudes from the angles bisectors. You can also cut off the altitudes so they don't stick out of the triangle. However sometimes altitudes may, if you drag a vertex, appear outside the triangle, a cut off altitude will disappear.



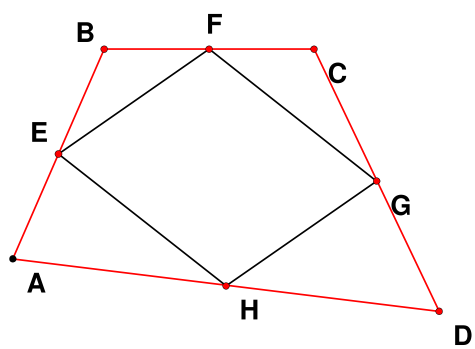
Further investigation: You could go back to the sketch with the medians, construct the centroid by highlighting two of the medians and using *Intersection* on the **Construct Menu.** Then the students could try to determine relationships among the distances from the centroid to the vertices and the midpoints already in the sketch. Another cool activity is to construct *Interiors* (**Construct Menu)** of the six triangles formed by the medians using different colors for adjacent triangles. For this activity it's best to hide the perpendicular bisectors that are already in the sketch. The students can be challenged to discover the relations among the six triangles. (They all have the same area.) Instead of telling them how to use *Area* on the **Measure Menu,** wait for them to ask, so as not to give away the secret.

Lab Exercise #3 A Parallelogram in a Quadrilateral

In Section 1.3, *Theorem 1.27* lists several properties of parallelograms that are equivalent to the definition and, a few pages later, is *Example 1.6* which discusses what happens when you join the midpoints of the sides of an arbitrary quadrilateral. Since this result is a surprise to many students, this example is good for a SketchPad discovery session. This can be done after covering *Theorem 1.27* and before covering *Example 1.6.* You can also show the class some SketchPad construction shortcuts. Here are the steps:

1) Plot four points with the *Point Tool.* Highlight them in the order you want them to be as the vertices of the quadrilateral (in either clockwise or counter clockwise order).
2) Clicking *Segments* on the **Construct Menu** will tell Sketchpad to connect the four points in that same order.
3) Now highlight the four segments (and no vertices). Using *Midpoints* on the **Construct Menu** will give you the four midpoints.
4) Mimic Step (1) to connect the four midpoints in cyclic fashion.

The students can now drag the four original vertices and discover the result. Next you can discuss how to use the items on the **Measure Menu** in conjunction with *Theorem 1.27* to see of their conjecture is correct.



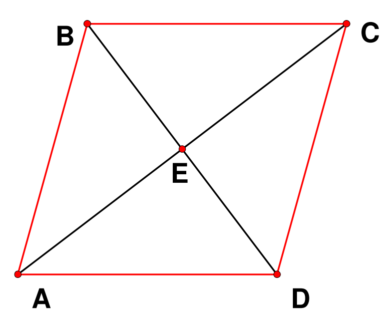
Lab Exercise #4 A Rhombus and Its Diagonals

Exercise 10 in Section 1.3 presents some problems in the same vein and be used to teach the students about using hidden objects to give the sketch invariant properties like, for example, creating a sketch that is a square and remains a square when the vertices are dragged.

Before starting Exercise 10a you can have the students construct a quadrilateral in which the diagonals bisect each other and are perpendicular as follows:

1) Construct any segment *AB* and its midpoint *M*.
2) Highlight the segment and *M* and construct the perpendicular at *M*.
3) To put two points *C* and *D* on the bisector so that *M* is the midpoint of *CD*:
   a) Highlight the perpendicular bisector and use *Construct Point* on the **Construct Menu.** Say the point is labeled *C*
   b) Highlight *M* and *C* (in that order) and construct *Circle by Center and Point.*
   c) Highlight the circle and the line *CM* and then construct *Intersection.* Label *D* as that point.
   d) Highlight the circle and hide it using *Hide* on the **Display Menu.**
   e) Construct the segment *CD* . Then *AB* and *CD* will remain perpendicular when the vertices are dragged. Also, *C* and *D* will stay on the circle even when it's hidden so that *M* will remain the midpoint of CD.

Now *A, B, C* and *D* can be cyclically connected by segments and the students should be able to guess a rhombus is formed.



This may be a good time to show how to use hidden circles to construct a square that remains a square while changing size during dragging.

1) Construct a segment *AB*
2) Construct lines that are perpendicular to *AB* at points *A* and *B*.
3) Construct a circle with center *A* and through *B*.
4) Construct a point *C* of intersection of the circle and the perpendicular at *A*.
5) Hide that perpendicular and the circle.
6) Construct the segment *CD*.

7) Construct a circle with center *B* and through *A*.
8) Construct the intersection point *D* of this circle and the perpendicular at *B*.
9) Hide the circle and the perpendicular and construct the segment *DB*
10) Finally, join *C* and *D* with a segment

Lab Exercise #5 Tools and The Pythagorean Theorem

A useful feature of SketchPad is the "*Tool*" feature which allows the user to automate constructions that are used repeatedly. A good exampl is the *Pythagorean Theorem* (Example 0.7) paired with Problem 0.3. The Pythagorean Theorem is so well known to students that it's not much good for a discovery session, but Problem 0.3 is.

The tool we are about to describe will **construct a square**, given the endpoints of one of the sides. Here are the steps:

1) Construct two points (say A and B) and the segment joining them. Nex, we construct another side of the square by rotating this side 90°.
2) To do this, double click the left end of the segment (let's assume it's A) to mark it as the center of rotation. You should see a fleeting bullseye around A to show it has been marked. Then highlight B and the segment to indicate that they are what is to be rotated. Now choose *Rotate* from the **Transform Menu.** A dialog box will appear and you can choose the angle of rotation. Since +90° (counterclockwise) is the default, you may merely click on rotate. A new side of the square will appear, with one end at A and a new point at the other end, named B′.
3) Repeat this process to construct the third side. That is, double click B′, highlight A and the segment B'A and the use *Rotate* on the **Transform Menu** again.
4) For the fourth side, double click the new vertex which will be A′, highlight B′ and A′B′ and rotate 90°.
5) Next highlight the vertices, in order, and use *Quadrilateral* on the **Construct Menu.** We do this so we can use *Area* on the **Measure Menu.** The interior of the triangle should be shaded.
6) Next we save these steps as a *Tool* so they can be repeated with a single key stroke:
    a) First highlight what needs to be there to start the construction of the square (the *Givens*). The Givens will be the two starting endpoints of the original segment. In this example, the Givens are the two points *A* and *B*.
    b) Now highlight what you want the tool to construct. In this example, we want the square and its interior to be constructed by the tool, so we highlight everything (*Ctrl-A* works).
    c) In the **Toolbox** click the *Tool* icon (double arrow, last item) then click *Create Tool.*
    d) In the dialog box, type a name for this tool (like **Square)** and check *Show Script View,* then close the dialog box. The steps of the *Square* script will appear in a new dialog box.

7) To use this tool with other sketches you can save this sketch and the embedded tool in C:\Program Files\Sketchpad\Tool Folder. If you do not have access to this folder (if you are using a computer in a shared lab) then you can create a Tool Folder on a portable memory device. See *Tool Folders* in SketchPad **Help.**

We can now use the tool to illustrate the Pythagorean Theorem. For a first try at tool usage we suggest not opening a new sketch but adding to the current sketch which already has to Square Tool on screen.

Constructing an arbitrary triangle: The students can see the relation between $a^2 + b^2$ and c^2 when the angles vary from 90° to less than 90° and more than 90°.

1) Construct a triangle with vertices *A, B* and *C* in counterclockwise order.
2) Highlight *B* and *A,* in that order. The '***Givens'*** in the **Script Box** should be highlighted at this point. If they aren't then either *A* or *B* aren't highlighted or something else is highlighted. Assuming the highlighting has gone as indicated, you can complete the square construction by clicking "*Apply All Steps"* in the script box or, if you prefer, click "*Next Step"* repeatedly. If the square does not appear outside the triangle then you clicked *A* and *B* in the wrong order.
3) To construct the second square, highlight *C* and *B* (in that order), apply the tool, highlight *A* and *C* (in that order) and apply the tool again. Finally, highlight *C* and *A* in that order and apply the tool a third time. The triangle should now have a square attached to each side and each square should be shaded. .
4) To illustrate the Pythagorean Theorem and related inequalities, click one square to highlight it and then use *Area* on the **Measure Menu.** The area will appear in a small Text Box. Display the areas of the other two squares in the same manner.
5) To see how these areas vary with the angle, highlight *A, C* and *B* in that order and then use *Angle* on the **Measure Menu.**
6) To display the quantity a^2+b^2, open the Calculator by clicking *Calculate* on the **Measure Menu.** A dialog box will open. Click on the displayed (text) area of the square opposite vertex *A,* click "^" on the calculator keypad, then "2" and "+" on the calculator keypad. Next click the displayed text area of the square opposite *B* followed by "^", "2". (^, +, 2 can also typed on your computer's keyboard). Finally clicking "OK" on the displayed calculator's keypad will display a text box containing a^2+b^2. In a similar fashion c^2 can be displayed. (c is the side of the square opposite the vertex *C*. Now the students can drag the vertices of the triangle to change its shape and observe what happens to the quantities $a^2 + b^2$ and $c^{2.}$

Next we can use the same tool in a discovery session for Problem 0.3. We suggest doing this construction on the same SketchPad page because the *Square Tool* is already there.

1) Use the Point Tool from the toolbox to plot four points and then highlight all four in cyclic order. Use *Segment* on the **Construct Menu** to construct the four sides of the quadrilateral.

2) Highlight the endpoints of one side of the quadrilateral. <u>*Common Error Alert:*</u> The second one you highlight will swing around the first in a clockwise manner, so make sure you highlight the two points so that you get a square that is outside the quadrilateral. (If not, you can undo the steps of the tool by pressing *Ctrl-Z* repeatedly.) Use the tool three more times to get a square attached to each of the four sides.
3) Next construct the midpoint of each square. To do this, highlight opposite corners of one of the squares, construct the segment joining these two and then construct its midpoint. Repeat this three more times so that each of the four attached squares has a midpoint. Label the midpoints C_1, C_2, C_3, and C_4. (To change the label, position the point so that becomes a finger pointing at the label you want to change and double click. In the resulting dialog box you can type your new label. To get C_1, type C[1])
4) See if your students can discover the relation between the lengths of the segments C_1C_3 and C_2C_4.