Errata

This document contains the most up to date set of known errors in *Closer and Closer: Introducing Real Analysis* by Carol S. Schumacher. There is a list of substantive errors at the beginning. A list of typographical errors is listed at the end of the document.

Last Updated: June 19, 2008

0.2 Functions

- Page 24—Problems 2(c) and 2(d) should read thus (I have underlined the words that need to be changed. The underline should not appear in the text.)

  (c) If $g \circ f$ is onto, then f is <u>onto</u>.

  (d) If $g \circ f$ is onto, then g is <u>onto</u>.

1.4 Least Upper Bound Axiom

- Page 60—Problem 3 is false, as stated. t must be non-negative.

  Original phrasing:

  Let $t \in \mathbb{R}$ and let $S \subset \mathbb{R}$ that is bounded above.

  Suggested rephrasing:

  Let $t \in \mathbb{R}^+$ and let $S \subset \mathbb{R}$ that is bounded above.

- Page 60—In problem 4, the sets S and T need to be non-empty. The problem should say, "Let S and T be non-empty subsets of $\mathbb{R}$ that are bounded above.

2.2 The Euclidean Metric on $\mathbb{R}^n$

- Page 65—Middle of the page. The definition of the metric on $\mathbb{R}^n$ is mistypeset as a fraction. It should read:

  $$d((a_1, a_2, \ldots, a_n), (b_1, b_2, \ldots, b_n)) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + \ldots + (a_n - b_n)^2}.$$

3.4 Sequences in $\mathbb{R}$

- Page 92— In problem 10, $\lim\limits_{n\to} c^{\frac{1}{n}} = 1$ should be $\lim\limits_{n\to\infty} c^{\frac{1}{n}} = 1$.

- Page 93—In problem 11, the reference to Excursion D.4.7 should really be a reference to Exercise D.4.7.

3.7 Open Sets, Closed Sets, and the Closure of a Set

- Page 99—In part 5 of Exercise 3.7.4 the statement "Let $x \in \overline{X}$" should, instead, be "Let $x \in X$."

4.3 Continuous Functions

- Page 114—The point a referred to in problem 7 must be a limit point of X, otherwise the limit is not defined.

  Original phrasing:

  ...Prove that f is continuous at $a \in X$ if and only if ...

  Should be:

  ...Prove that f is continuous at a limit point a of X if and only if ...

4.4 Uniform Continuity

- Page 116—For problem 6(c). Add the parenthetical statement

  > (Assume for now that the difference of two continuous, real-valued functions is continuous. This will be proved in Section 5.3.)

  at the end of the statement of the problem.

5.1 Limits, Continuity, and Order

- Page 122—Second paragraph after **Some Useful Special Cases**. The sentence with the bad reference (??) should be: "Corollary 5.1.9 is a special case of Theorem 5.1.1."

5.3 Limits, Continuity, and Arithmetic

- Page 127—In theorem 5.3.1(4), the statement reads "Assume $g(x) \neq 0$ on some interval containing a ..." it should, instead, be "Assume $g(x) \neq 0$ on some open set containing a ..."

7.1 Compact Sets

- Page 143—In part (b) of problem 17. The set X should be

$$X = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1, x > 0 \text{ and } y > 0\}.$$

8.1 Connected Sets

- Page 153—Problem 1. The problem reads,

  > Prove the IVT for a continuous function $f : [a, b] \to \mathbb{R}$ as follows. Suppose that γ is between $f(a)$ and $f(b)$. Let $c = \sup\{x \in [a, b] : f(x) \leq \gamma\}$. Show that $f(c) = \gamma$.

  It should, instead, read

  > Prove the IVT for a continuous function $f : [a, b] \to \mathbb{R}$ as follows. Suppose that $f(a) \leq \gamma \leq f(b)$. Let $c = \sup\{x \in [a, b] : f(x) \leq \gamma\}$. Show that $f(c) = \gamma$. Modify the argument for the case when $f(b) \leq \gamma \leq f(a)$.

9.2 The Derivative

- Page 163— In the first line of the second paragraph contained in the box, "$y \to 0$" should be "$y \to x$."
- Page 163—In the last line in the box "the expression" should be "the expression in Theorem 9.2.2."

9.7 Polynomial Approximation and Taylor's Theorem

- Page 184—In the proof of Theorem 9.7.1, third line from the end. The line reads: "But $A''(x) = f'' - M$, so $A''(c) = 0$ which implies that $M = f''(c)$"

  It should, instead, read "But $A''(x) = f'' - M$. So $A''(c) = 0$ implies that $M = f''(c)$"

10.1 Iteration and Fixed Points

- Page 197—Problem 8, second line reads " $\ldots 1 \leq k \leq 3 \ldots$." It should, instead, read " $\ldots 1 < k < 3$."

11.4 Families of Riemann Sums

- Page 223—Last line of the page reads:

  $$N^*(z_j - z_{i-1}) \text{ where } N^* = \sup\{f(x) : x \in [z_i, z_j]\}.$$

  It should read, instead,

  $$N^*(z_j - z_i) \text{ where } N^* = \sup\{f(x) : x \in [z_i, z_j]\}.$$

11.5 Existence of the Integral

- Page 228—The definition of the function in Example 11.5.3 needs to be modified slightly. Add "or 0" in the first line of the definition:

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational or } 0 \\ \frac{1}{q} & \text{if } x \text{ is rational and } x = \frac{p}{q} \end{cases}$$

- Page 228—Theorem 11.5.4. The third sentence reads "If $f : K \to \mathbb{R}$ is a function, then the integral $\int_a^b f \ldots$." It should, instead, read "If $f : K \to \mathbb{R}$ is a function and $a < c < b$, then the integral $\int_a^b f \ \ldots$"

- Page 232—The expression at the bottom of the page reads.

$$\left| \mathcal{R}(f, P) - \left(\int_a^c + \int_c^b f \right) \right|.$$

  It should read, instead,

$$\left| \mathcal{R}(f, P) - \left(\int_a^c f + \int_c^b f \right) \right|.$$

- Page 233—Problem 4(c) currently reads: "Now use the result from parts (a) and (b) to remove the restriction that $a < b < c$. (You may need to break this into several cases.)" It should, instead say "Assume any two of the three integrals $\int_a^c f$, $\int_c^b f$ and $\int_a^b f$ exist. Use the result from parts (a) and (b) to remove the restriction that $a < c < b$. (You may need to break this into several cases.)"

- Page 233—The definition of the function in Problem 5 needs to be modified slightly. Add "or 0" in the first line of the definition:

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational or } 0 \\ \frac{1}{q} & \text{if } x \text{ is rational and } x = \frac{p}{q} \end{cases}$$

  And the word "non-zero" needs to be added to the statement in part (a): "Show that f is discontinuous at every non-zero rational number."

12.2 Uniform Convergence

- Page 245—In problem 12, the next to the last line before part (a). " $\ldots$ that $\lim_{m\to\infty} = f(x)$" should, instead, read "$\ldots$ that $\lim_{m\to\infty} f(x_m) = f(x)$."

13.1 What Are We Studying

- Page 259—Theorem 13.1.2(1). The second line reads "$\ldots \lim_{k\to\infty} \mathbf{f}(\mathbf{y}_n) = \mathbf{b}$ if and only if for each $i = 1, 2, \ldots, m$, $\lim_{k\to\infty} f_i(\mathbf{y}_n) = b_i$." the subscripts n should, instead be k's. The line should read "$\ldots \lim_{k\to\infty} \mathbf{f}(\mathbf{y}_k) = \mathbf{b}$ if and only if for each $i = 1, 2, \ldots, m$, $\lim_{k\to\infty} f_i(\mathbf{y}_k) = b_i$."

13.3 Analysis in Linear Spaces

- Page 265—Theorem 13.3.9(3). The linear transformation referred to in this part of the problem needs to be one-to-one. The problem should read: "Suppose that $\mathbf{L}$ is one-to-one. then the set $\{\mathbf{L}(\mathbf{e}_i)\}$, the image of the standard basis, is linearly independent in $\mathbb{R}^m$."

- Page 267-268—In Theorem 13.3.20. Because n is the dimension of the space in this problem, every subscript n should be changed to an i. The theorem should read:

  Let $(\mathbf{x}_i)$ and $(\mathbf{y}_i)$ be sequences in $\mathbb{R}^n$ converging to $\mathbf{x}$ and $\mathbf{y}$, respectively. Let (t_i) be a sequence in $\mathbb{R}$ that converges to a scalar t. Let k be an arbitrary scalar. Prove the following facts:

  1. $(k\mathbf{x}_i)$ converges to $k\mathbf{x}$.
  2. $(t_i\mathbf{x}_i)$ converges to $t\mathbf{x}$.
  3. $(\mathbf{x}_i + \mathbf{y}_i)$ converges to $\mathbf{x} + \mathbf{y}$.
  4. $(\mathbf{x}_i \cdot \mathbf{y}_i)$ converges to $\mathbf{x} \cdot \mathbf{y}$.

- Page 270—In Problem 10—Because n is the dimension of the space in this problem, every subscript n should be changed to an i. The problem should read

  Let $(\mathbf{x}_I)$ be a sequence in $\mathbb{R}^n$ and let (t_i) be a sequence of scalars.

  (a) Suppose that $(\mathbf{x}_i)$ converges to $\mathbf{0}$ and that (t_i) is bounded in $\mathbb{R}$. Prove that $(t_i\mathbf{x}_i)$ converges to $\mathbf{0}$.

  (b) Suppose that (t_i) is a sequence in $\mathbb{R}$ that converges to 0, and $(\mathbf{x}_i)$ is a bounded sequence in $\mathbb{R}^n$. Prove that $(t_i\mathbf{x}_i)$ converges to $\mathbf{0}$.

- Page 271—Problem 12, first line—"established in Lemma 13.3.21 ..." should, instead, read, "...established in Corollary 13.3.23."

- Page 271—In problem 15(a). There is a typographical error in the description of $\mathrm{B}_r(\mathbf{x})$. It reads "$\mathbf{x} + s\mathbf{u} : 0 \le s \le r \ldots$." It should, instead, read "$\mathbf{x} + s\mathbf{u} : 0 \le s < r \ldots$."

- Page 271—In problem 16, second line: it reads " ... if and only if $\mathbf{L}(\mathbf{e}_i) = \mathbf{S}(\mathbf{e}_i)$ ...." It should, instead, read "... if and only if $\mathbf{T}(\mathbf{e}_i) = \mathbf{S}(\mathbf{e}_i)$ ...."

13.4 Local Linear Approximation

- Page 277—In Theorem 13.4.8—last line before the equation at the end reads "it follows that for $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$, ..." The last quantification is unnecessary. It should read "it follows that for $i = 1, 2, \ldots, n$, ..."
- Page 279—In Theorem 13.4.12, in "Step 1." The very beginning reads "Define $r : E \to \mathbb{R}$ ...." It should, instead, say "Define $e : E \to \mathbb{R}$ ...."
- Page 283—Problem 8. The displayed equation at the very end should read,

$$f(\mathbf{x}) = \nabla f(\mathbf{a}) \cdot (\mathbf{x} - \mathbf{a}) + f(\mathbf{a}) + r(\mathbf{x}), \qquad \text{and } \lim_{\mathbf{x} \to \mathbf{a}} \frac{|r(\mathbf{x})|}{\|\mathbf{x} - \mathbf{a}\|} = 0.$$

- Page 284—Problem 13. The function is incorrect. It should, instead, be

$$f(x, y) = \begin{cases} \dfrac{yx + 2xy^2}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}.$$

- Page 286—The displayed expression near the bottom of the page is off by a minus sign.

$$D_{\mathbf{1}} D_{\mathbf{2}} f(\mathbf{a}) \approx \frac{\frac{f(a_1, a_2+k) - f(a_1, a_2)}{k} - \frac{f(a_1+h, a_2+k) - f(a_1+h, a_2)}{k}}{h}$$

should, instead, be

$$D_{\mathbf{1}} D_{\mathbf{2}} f(\mathbf{a}) \approx \frac{\frac{f(a_1+h, a_2+k) - f(a_1+h, a_2)}{k} - \frac{f(a_1, a_2+k) - f(a_1, a_2)}{k}}{h}.$$

Excursion D.4 Some Important Special Sequences

- Page 317—Middle of the page (Step 2. in the proof sketch for Theorem D.4.5). "Excursion 3" should instead be "Excursion C."

Excursion F.1 Double Sequences and Convergence

- Page 328—In definition F.1.6, 4th line reads, "... $N \in \mathbb{N}$ such that for all $m > N$ and all $N \in \mathbb{N}$ ...." It should, instead, read " ... $N \in \mathbb{N}$ such that for all $m > N$ and all $n \in \mathbb{N}$ ... "

- Page 329—The metric space in Theorem F.1.7 needs to be complete. In other words, the hypothesis should read "Let X be a complete metric space, ..."

Excursion H.1 Series of Real Numbers

- Page 336—Theorem H.1.5 in the last line before the displayed inequality, "$n > m > N$" should, instead, read "$n \geq m > N$."

Excursion H.4 Rearranging the Terms of a Series

- Page 353—As stated in Lemma H.4.5, the last word in problem 2 should be "diverge" not "converge."

Excursion I.1 Regular Riemann Sums

- Page 359—The second displayed expression reads

$$\sum_{i=0}^{n-1} f(x_i)(x_i - x_{i-1}).$$

It should, instead, be

$$\sum_{i=1}^{n} f(x_{i-1})(x_i - x_{i-1})$$

Excursion J.1 Power Series

- Page 366—The second and fourth power series given in Exercise J.1.6 should start at $n = 1$:

$$\sum_{n=0}^{\infty} \frac{3}{n^2}(x-5)^n \text{ should instead be } \sum_{n=1}^{\infty} \frac{3}{n^2}(x-5)^n.$$

$$\sum_{n=0}^{\infty} \frac{1}{n}(x-5)^n \text{ should instead be } \sum_{n=1}^{\infty} \frac{1}{n}(x-5)^n.$$

Excursion M.4 The Inverse Function Theorem

- Page 407—in problem 2, the second line, "The reverse is also possible" should instead read "The reverse is also possible provided that we assume all of the partial derivatives of $\mathbf{F}$ exist and are continuous." The next to the last line of the problem should read "...indeed, equivalent under the hypothesis that all partial derivatives exist and are continuous, assume ..."

Excursion N.3 The Stone-Weierstrass Theorem

- Page 419—in step 2 of problem 4(b). The text should read: "Let $x \in [0,1]$. Notice that if $f(x) = x$, then $f(x) = f(x) - (f(x))^2 + x^2$. In fact, $f(x) = x$ is the unique non-negative fixed point for the function $F : C[0,1] \to C[0,1]$ given by $F(f) = f - f^2 + q$ where q is the quadratic function $q(x) = x^2$ on $[0,1]$."
- Page 420—Steps 3 and 4 should be reversed.

Excursion O.2 Picard Iteration

- Page 426—In problem 1, U must be convex. The problem should read, "Let $U \subseteq \mathbb{R}^2$ be convex, and let $f : U \to \mathbb{R} \ldots$"
- Page 426—In problem 2. The displayed equation should match the corresponding equation on the previous page:

$$F(y)(t) = x_0 + \int_{t_0}^{t} f(u, y(u))\, du.$$

Excursion O.3 Systems of Equations

- Page 430—In problem 5. The constant α mentioned in the result should be $\alpha = \min\left\{r, \frac{m}{nM}\right\}$.

Less important errors (more in the way of typos.)

- Page 80—In problem 11, the $\overline{X}$ should just be X.
- Page 108—The word "appoaches" in Theorem 4.2.3 should, instead, be "approaches."
- Page 112—There should be a period at the end of the displayed equation on the very last line.
- Page 132—The square brackets around the Hint at the end of problem 3(b) should, instead, be parentheses.
- Page 142—In problem 11, third line: "susequences" should be "subsequences."
- Page 160—In the last paragraph, end of the first line, there is a comma after the word countably many. This comma shouldn't be there.
- Page 212—In Theorem 11.2.3, in the second line we see "…that is a Riemann integrable on …". It should read, instead,"…that is Riemann integrable on …."

- Page 212—In Theorem 11.2.3, in the second line we see "...that is a Riemann integrable on ...". It should read, instead,"...that is Riemann integrable on ...."
- Page 244—The last line of the page needs a space between "functions" and "on."
- Page 245—In problem 12, third line. The sentence at the end of the line that begins "The for all ..." should instead begin with "Then for all ...."
- Page 252—The first line of Corollary 12.4.5 ends in the word "is" and should, instead, end in the word "are."
- Page 277—Theorem 13.4.8: The function referred to is a vector-valued function. Thus in lines 2 and 3, f should instead be $\mathbf{f}$.
- Page 282—Problem 3. The function f mentioned in the first line should, instead be $\mathbf{f}$, as it is a vector-valued function.
- Page 330—In problem 1, there should be a comma between "*non*-convergent" and "bounded."
- Page 305—In Theorem C.1.2, the first line should read "Let a be a positive real number. Let r and s ..."
- Page 361—In problem 2, "Reimann" should, instead, be "Riemann."