Errata

This document contains the most up to date set of known errors in *Closer and Closer: Introducing Real Analysis* by Carol S. Schumacher. There is a list of substantive errors at the beginning. A list of typographical errors is listed at the end of the document.

Last Updated: January 24, 2008

- Page 24—Problems 2(c) and 2(d) should read thus (I have underlined the words that need to be changed. The underline should not appear in the text.)

  (c) If $g \circ f$ is onto, then f is <u>onto</u>.

  (d) If $g \circ f$ is onto, then g is <u>onto</u>.

- Page 60—Problem 3 is false, as stated. t must be non-negative.

  Original phrasing:

  Let $t \in \mathbb{R}$ and let $S \subset \mathbb{R}$ that is bounded above.

  Suggested rephrasing:

  Let $t \in \mathbb{R}^+$ and let $S \subset \mathbb{R}$ that is bounded above.

- Page 65—Middle of the page. The definition of the metric on $\mathbb{R}^n$ is mistypeset as a fraction. It should read:

  $$d((a_1, a_2, \ldots, a_n), (b_1, b_2, \ldots, b_n)) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + \ldots + (a_n - b_n)^2}.$$

- Page 92— In problem 10, $\lim_{n\to} c^{\frac{1}{n}} = 1$ should be $\lim_{n\to\infty} c^{\frac{1}{n}} = 1$.

- Page 93—In problem 11, the reference to Excursion D.4.7 should really be a reference to Exercise D.4.7.

- Page 99—In part 5 of Exercise 3.7.4 the statement "Let $x \in \overline{X}$" should, instead, be "Let $x \in X$."

- Page 114—The point a referred to in problem 7 must be a limit point of X, otherwise the limit is not defined.

  Original phrasing:

  ... Prove that f is continuous at $a \in X$ if and only if ...

  Suggested rephrasing:

  ... Prove that f is continuous at a limit point a of X if and only if ...

- Page 116—For problem 6(c). Add the parenthetical statement

  > (Assume for now that the difference of two continuous, real-valued functions is continuous. This will be proved in Section 5.3.)

  at the end of the statement of the problem.

- Page 122—Second paragraph after **Some Useful Special Cases**. The sentence with the bad reference (??) should be:

  > Corollary 5.1.9 is a special case of Theorem 5.1.1.

- Page 127—In theorem 5.3.1(4), the statement reads "Assume $g(x) \neq 0$ on some interval containing $a \ldots$" it should, instead, be "Assume $g(x) \neq 0$ on some open set containing $a \ldots$"

- Page 143—In part (b) of problem 17. The set X should be

$$X = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1, x > 0 \text{ and } y > 0\}.$$

- Page 153—Problem 1. The problem reads,

  > Prove the IVT for a continuous function $f : [a, b] \to \mathbb{R}$ as follows. Suppose that γ is between $f(a)$ and $f(b)$. Let $c = \sup\{x \in [a, b] : f(x) \leq \gamma\}$. Show that $f(c) = \gamma$.

  It should, instead, read

  > Prove the IVT for a continuous function $f : [a, b] \to \mathbb{R}$ as follows. Suppose that $f(a) \leq \gamma \leq f(b)$. Let $c = \sup\{x \in [a, b] : f(x) \leq \gamma\}$. Show that $f(c) = \gamma$. Modify the argument for the case when $f(b) \leq \gamma \leq f(a)$.

- Page 163— In the first line of the second paragraph contained in the box, "$y \to 0$" should be "$y \to x$."

- Page 163—In the last line in the box "the expression" should be "the expression in Theorem 9.2.2."

- Page 223—Last line of the page reads:

$$N^*(z_j - z_{i-1}) \text{ where } N^* = \sup\{f(x) : x \in [z_i, z_j]\}.$$

  It should read, instead,

$$N^*(z_j - z_i) \text{ where } N^* = \sup\{f(x) : x \in [z_i, z_j]\}.$$

- Page 228—The definition of the function in Example 11.5.3 needs to be modified slightly. Add "or 0" in the first line of the definition:

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational or } 0 \\ \frac{1}{q} & \text{if } x \text{ is rational and } x = \frac{p}{q} \end{cases}$$

- Page 228—Theorem 11.5.4. The third sentence reads "If $f : K \to \mathbb{R}$ is a function, then the integral $\int_a^b f \ldots$." It should, instead, read "If $f : K \to \mathbb{R}$ is a function and $a < c < b$, then the integral $\int_a^b f \ \ldots$"

- Page 233—Problem 4(c) currently reads: "Now use the result from parts (a) and (b) to remove the restriction that $a < b < c$. (You may need to break this into several cases.)" It should, instead say "Assume any two of the three integrals $\int_a^c f$, $\int_c^b f$ and $\int_a^b$ exist. Use the result from parts (a) and (b) to remove the restriction that $a < b < c$. (You may need to break this into several cases.)"

- Page 233—The definition of the function in Problem 5 needs to be modified slightly. Add "or 0" in the first line of the definition:

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational or } 0 \\ \frac{1}{q} & \text{if } x \text{ is rational and } x = \frac{p}{q} \end{cases}$$

  And the word "non-zero" needs to be added to the statement in part (a): "Show that f is discontinuous at every non-zero rational number."

- Page 232—The expression at the bottom of the page reads.

$$\left| \mathcal{R}(f, P) - \left(\int_a^c + \int_c^b f \right) \right|.$$

  It should read, instead,

$$\left| \mathcal{R}(f, P) - \left(\int_a^c f + \int_c^b f \right) \right|.$$

- Page 233—In item (c) of problem 4, the second line reads "... that $a < b < c$." It should read, instead, "... that $a < c < b$".

- Page 259—Theorem 13.1.2(1). The second line reads " $\ldots \lim_{k\to\infty} \mathbf{f}(\mathbf{y}_n) = \mathbf{b}$ if and only if for each $i = 1, 2, \ldots, m$, $\lim_{k\to\infty} f_i(\mathbf{y}_n) = b_i$." The subscripts n should instead be k's. The line should read "$\ldots \lim_{k\to\infty} \mathbf{f}(\mathbf{y}_k) = \mathbf{b}$ if and only if for each $i = 1, 2, \ldots, m$, $\lim_{k\to\infty} f_i(\mathbf{y}_k) = b_i$."

- Page 286—The displayed expression near the bottom of the page is off by a minus sign.

$$D_{\mathbf{1}}D_{\mathbf{2}}f(\mathbf{a}) \approx \frac{\frac{f(a_1,a_2+k)-f(a_1,a_2)}{k} - \frac{f(a_1+h,a_2+k)-f(a_1+h,a_2)}{k}}{h}$$

  should, instead, be

$$D_{\mathbf{1}}D_{\mathbf{2}}f(\mathbf{a}) \approx \frac{\frac{f(a_1+h,a_2+k)-f(a_1+h,a_2)}{k} - \frac{f(a_1,a_2+k)-f(a_1,a_2)}{k}}{h}.$$

- Page 317—Middle of the page (Step 2. in the proof sketch for Theorem D.4.5). "Excursion 3" should instead be "Excursion C."

- Page 336—Theorem H.1.5 in the last line before the displayed inequality, "$n > m > N$" should, instead, read "$n \geq m > N$."

- Page 353—As stated in Lemma H.4.5, the last word in problem 2 should be "diverge" not "converge."

- Page 359—The second displayed expression reads

$$\sum_{i=0}^{n-1} f(x_i)(x_i - x_{i-1}).$$

  It should, instead, be

$$\sum_{i=1}^{n} f(x_{i-1})(x_i - x_{i-1})$$

- Page 366—The second and fourth power series given in Exercise J.1.6 should start at $n = 1$:

$$\sum_{n=0}^{\infty} \frac{3}{n^2}(x-5)^n \text{ should instead be } \sum_{n=1}^{\infty} \frac{3}{n^2}(x-5)^n.$$

$$\sum_{n=0}^{\infty} \frac{1}{n}(x-5)^n \text{ should instead be } \sum_{n=1}^{\infty} \frac{1}{n}(x-5)^n.$$

Less important errors (more in the way of typos.)

- Page 80—In problem 11, the $\overline{X}$ should just be X.

- Page 108—The word "appoaches" in Theorem 4.2.3 should, instead, be "approaches."

- Page 112—There should be a period at the end of the displayed equation on the very last line.

- Page 132—The square brackets around the Hint at the end of problem 3(b) should, instead, be parentheses.
- Page 142—In problem 11, third line: "susequences" should be "subsequences."
- Page 160—In the last paragraph, end of the first line, there is a comma after the word countably many. This comma shouldn't be there.
- Page 212—In Theorem 11.2.3, in the second line we see "...that is a Riemann integrable on ...". It should read, instead,"...that is Riemann integrable on ...."
- Page 212—In Theorem 11.2.3, in the second line we see "...that is a Riemann integrable on ...". It should read, instead,"...that is Riemann integrable on ...."
- Page 244—The last line of the page needs a space between "functions" and "on."
- Page 252—The first line of Corollary 12.4.5 ends in the word "is" and should, instead, end in the word "are."
- Page 330—In problem 1, there should be a comma between "*non*-convergent" and "bounded."
- Page 361—In problem 2, "Reimann" should, instead, be "Riemann."