Using a computer or standard approximation techniques such as Newton's method to solve this equation for λ, we find the value

$$\lambda \approx 0.8003.$$

Having thus determined all constants that appear in (67.15), we obtain

$$f(x) \approx 1.6317 \cosh(0.6129x) - 5.2595.$$

Furthermore, according to (66.22) the force that the rope exerts at the point of attachment $(d/2, 0)$ is

$$\mathbf{F} = -\lambda \begin{pmatrix} 1 \\ f'(d/2) \end{pmatrix} \approx \begin{pmatrix} -0.8003 \\ -2.4524 \end{pmatrix},$$

and the magnitude of this force is

$$\|\mathbf{F}\| \approx \sqrt{0.8003^2 + 2.4524^2} \approx 2.5797 \, \frac{kg \cdot m}{s^2}.$$

As a matter of course, the same value we also find for the magnitude of the force at the second point of attachment $(-d/2, 0)$.

Remark. The absolute value of the vertical force component at each of the points of attachment $(\pm d/2, 0)$ is equal to half the rope's total weight because, in denoting by m the rope's total mass, it follows that

$$\left|\lambda f'\left(\pm\frac{d}{2}\right)\right| = \left|\lambda \sinh\left(\pm\frac{\delta g d}{2\lambda}\right)\right| = \lambda \sinh\left(\frac{\delta g d}{2\lambda}\right) = \frac{\delta L g}{2} = \frac{mg}{2}.$$

67.31. Exercise. Find the function that describes the shape of a rope of mass $2\,kg$ and length $L = 15\,m$ under the assumption that the distance between its horizontally attached endpoints is $d = 7\,m$. Then use your result to determine the magnitude of the force that the rope exerts at the points of attachment.

Polar Graphs

Apart from graphs in ordinary xy-coordinates, we commonly also encounter *polar graphs* that are described by a function $r : [\alpha, \beta] \to \mathbb{R}$ which assigns to a given angle θ a distance $r(\theta)$ measured relative to the origin in a polar coordinate system. In other words, the polar graph described by r consists of the points $(\theta, r(\theta))$ plotted in a polar coordinate system for values of θ ranging from α to β. Given the transformation equations in Appendix A, the corresponding parameterization in xy-coordinates is easily seen to be

$$\mathbf{r}(\theta) = \begin{pmatrix} x(\theta) \\ y(\theta) \end{pmatrix} = \begin{pmatrix} r(\theta)\cos(\theta) \\ r(\theta)\sin(\theta) \end{pmatrix}.$$

Since

$$\begin{aligned} \|\mathbf{r}'(\theta)\| &= \sqrt{x'(\theta)^2 + y'(\theta)^2} = \sqrt{(r'(\theta)\cos(\theta) - r(\theta)\sin(\theta))^2 + (r'(\theta)\sin(\theta) + r(\theta)\cos(\theta))^2} \\ &= \sqrt{(r'(\theta)^2 + r(\theta)^2)(\cos^2(\theta) + \sin^2(\theta))} = \sqrt{r'(\theta)^2 + r(\theta)^2}, \end{aligned}$$

it follows that the path length of $\mathbf{r}$ is given by the equation

$$\boxed{L = \int_\alpha^\beta \sqrt{r'(\theta)^2 + r(\theta)^2}\, d\theta.} \tag{67.17}$$

67.32. Example. Let us consider the polar graph of the function $r(\theta) := 1 + \cos(\theta)$ for $\theta \in [0, 2\pi]$. The curve described by the corresponding parameterization

$$\mathbf{r}(\theta) = \begin{pmatrix} (1 + \cos(\theta))\cos(\theta) \\ (1 + \cos(\theta))\sin(\theta) \end{pmatrix}$$