Since f is easily seen to have a maximum at this critical point, we may infer that the storage capacity of a CAV disc is maximal when the inner radius of the annular storage region is half the outer radius.

12.8. Exercise. Use the second derivative test to verify that f assumes a maximum at $r = R/2$.

12.9. Exercise. Find the maximal storage capacity of a CAV disc with $R = 58\,mm$ for an infrared and a blue-light laser.

Additional Exercises

12.10. A fence is to be built at minimal cost around a plot of land consisting of a rectangle joined with two semidiscs at the sides (see Figure 12.10). How would you choose the radius of the semicircular discs if the fence is to enclose a total area of $100\,m^2$, and if the price per meter of fence is three times higher for the semicircular sides than for the straight horizontal sides?

Figure 12.10: a rectangle joined with two semidiscs.

12.11. What is the maximal area of a rectangle inscribed in the region bounded by the lines $x = 0$ and $y = 1$ and the curve $y = x^3$ as shown in Figure 12.11?

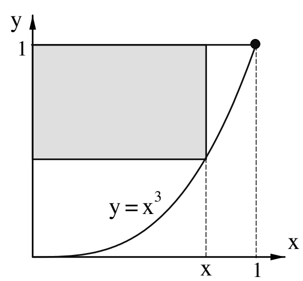

Figure 12.11: a rectangle inscribed in a parabolic region.

12.12. What is the maximal area of a triangle inscribed in a circle of radius 1 in the position shown in Figure 12.12?

12.13. Find the minimal distance from the line given by the equation $3x + 4y = 10$ to the origin $(0, 0)$, and determine the coordinates of the point P on the line that is closest to the origin.

12.14. Repeat Exercise 12.13 for the line given by the equation $x + 2y = 4$ and the point $(3, 3)$ (in place of $(0, 0)$).

12.15. Find the minimal distance from the graph of the function $f(x) := 2x^2 - 3x - 36/7$ to the origin $(0, 0)$, and determine the coordinates of the point P on the graph of f that is closest to the origin. *Hint.* You will encounter a cubic equation for which a simple integer solution can be guessed.

12.16. Repeat Exercise 12.15 for $f(x) := 5 - x^2$ and the point $(-3, 2)$ (in place of the origin $(0, 0)$).

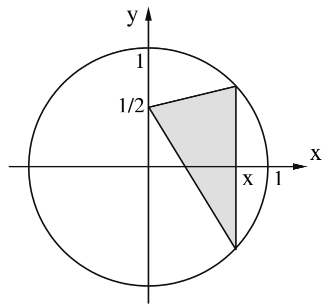

Figure 12.12: a triangle inscribed in a circle.

12.17. Find the point on the parabola $y = x^2 + 2$ that is closest to the line described by the equation $2x - y = 0$.

12.18. A billboard 54 feet wide is perpendicular to a straight road and is 18 feet east of the nearest point on the road. A car approaches the billboard from the south. From what point does a passenger in the car see the billboard at the widest angle?

12.19. How would you choose the height and radius of a $12\,oz$ ($= 355\,ml = 355\,ccm$) cylindrical aluminum can (including top and bottom) if you wanted to minimize the amount of aluminum used.

12.20. A taxi driving at a constant speed of $60\,mph$ gets 30 miles per gallon of fuel, and the price of fuel is $1.85 per gallon. Each increase in speed by 2 mph reduces the fuel efficiency by 0.5 miles per gallon. Other costs of running the taxi amount to $2.35 per hour. Assuming the taxi is to travel a given distance D, you are to determine what constant speed will minimize the total cost of operating the taxi?

12.21. A man has parked his car 500 meters down stream on the opposite shore of a river that is 200 meters wide. Assuming that he can swim at a speed of $1.5\,km/h$ and walk at a pace of $6\,km/h$, where along the opposite shore should he leave the water if he wishes to minimize the total time needed to get back to his car, and what actually is the minimal time?

12.22. The hourly cost of fuel in operating a train is proportional to the square of its speed, and for a speed of $40\,km/h$ the cost is $200. Other fixed expenses amount to $800 per hour. Assuming the train is to travel a distance D, you are to find the speed at which it should run to minimize the total cost.